Special Relativity

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Saturday Morning Physics

February 1st, 2020

Fermilab

Rewind back to the 1890s

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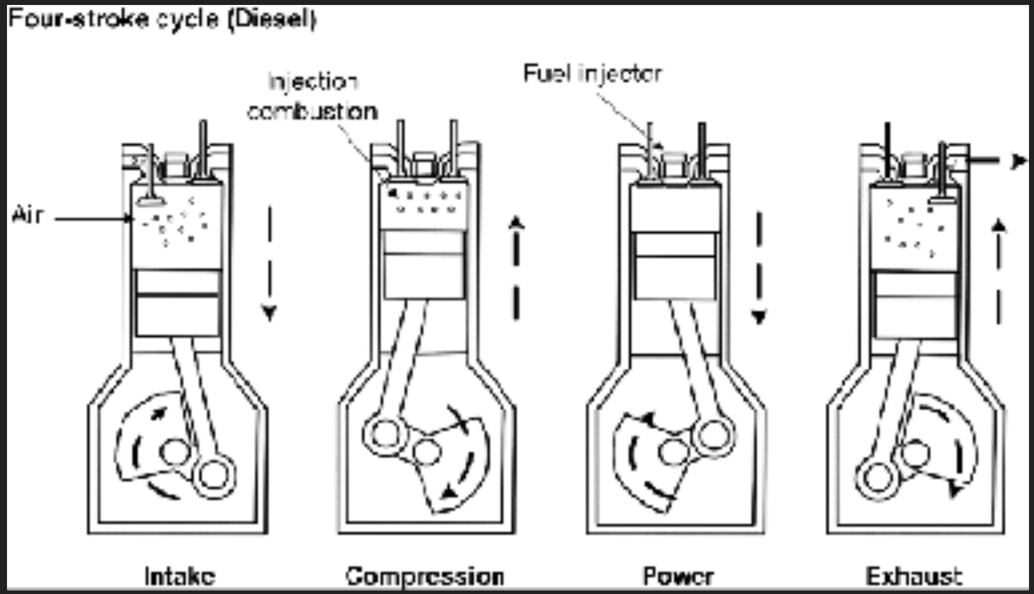
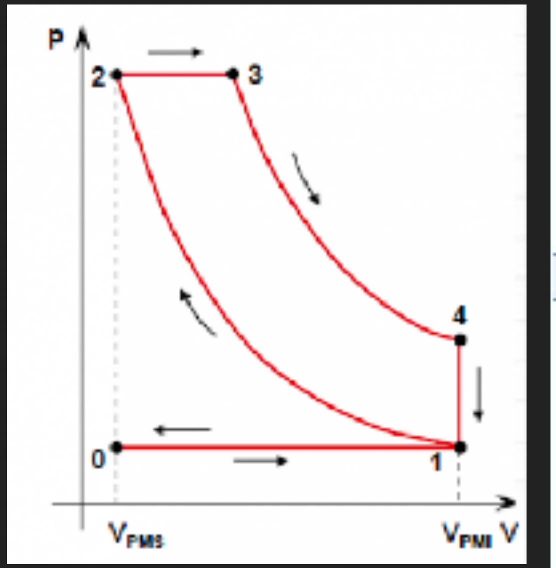
- Thermodynamics (1700s/1800s): contributions of many physicists. Determined how heat transfer works, led to the development of engines, etc.





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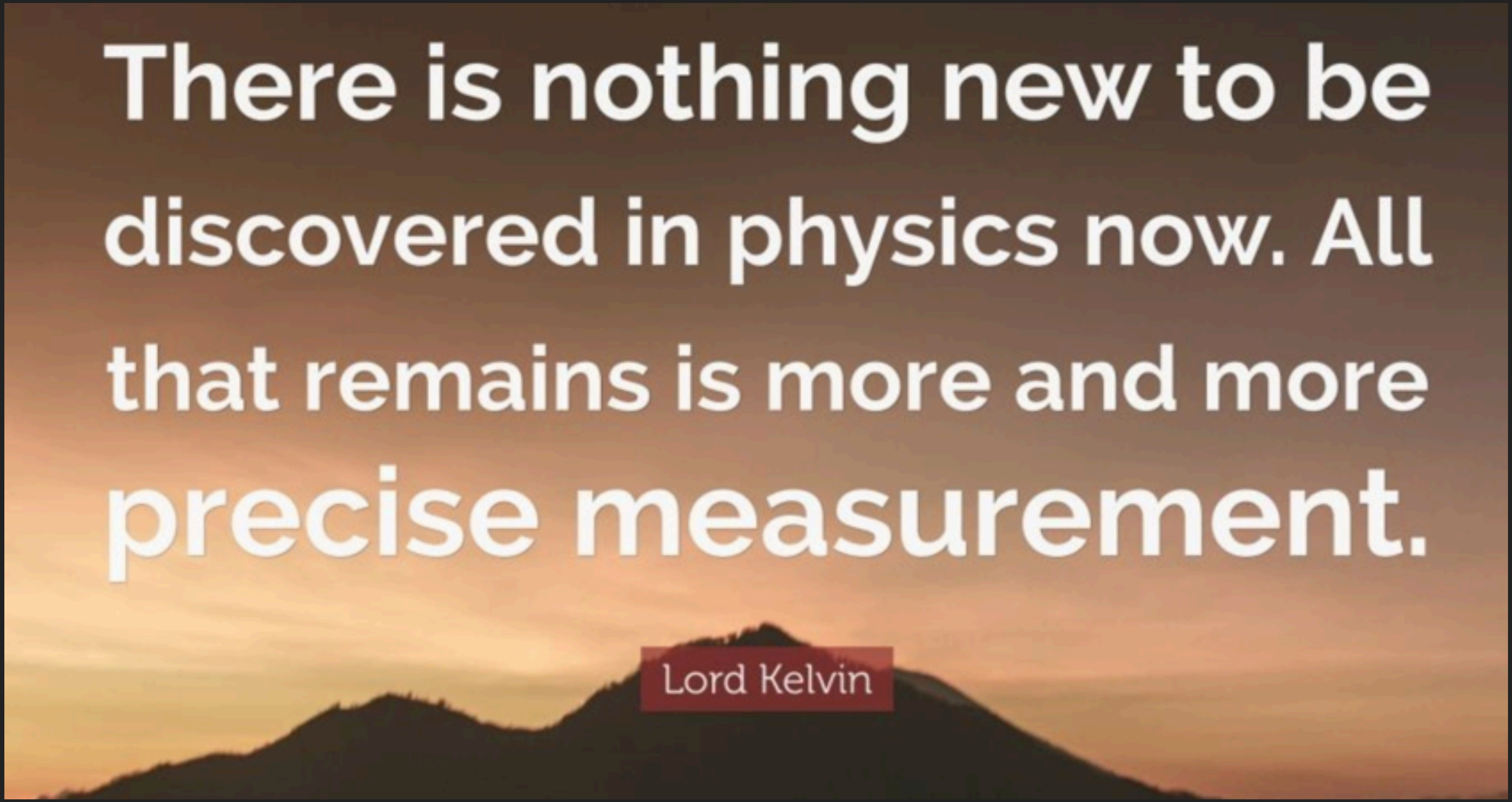
- Thermodynamics (1700s/1800s): contributions of many physicists. Determined how heat transfer works, led to the development of engines, etc.
- Electricity & Magnetism (1700s/1800s): a collection of newly-discovered laws were determined to be related, brought together as “Maxwell’s Equations”

In terms of describing what we see around us, and how we interact with the world, we have a pretty good handle on everything!



This is a VERY famous quote, often attributed to Lord Kelvin (one of the titans of Thermodynamics). Apparently, it's misattributed, but it was a common sentiment of the late 1800s.

A closer look at Maxwell's Equations

Name	Differential equations
Gauss's law	$\nabla \cdot \mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$

- In more straightforward terms:
  - Gauss's Law: The strength of an electric field leaving a surface is related to the amount of charge inside that surface.
  - GL for Magnetism: There's no such thing as a magnetic "charge"
  - Maxwell-Faraday: The strength of an electric field going around a loop is related to a changing magnetic field going through that loop.
  - Ampère's Law: The strength of a magnetic field around a loop is related to the current in that loop *and* the change of an electric field going through that loop.

An EVEN CLOSER look…

$$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

This number "c" here has units of speed, and is numerically about 300,000,000 meters per second.

This implies that the relationships between electric and magnetic fields depends on a "reference frame". Basically, what speed should enter the equations? The speed of light according to me? The speed of light according to someone else who's moving?

Brief Aside: Galilean Relativity

What do we mean by "Relativity"?

- When we say "relativity" or "relative motion", all we mean is what one person (observer) sees happening compared to another person.

- For instance, two people at rest in a room will agree on the speed of a bird flying by, but someone walking through the room will see the bird flying faster or slower.

Relative Velocities

- You are standing in the crowd at a track meet, watching your two friends Alice and Bob sprint by. In your “reference frame”, Alice is running 10 miles per hour to the right, and Bob is running 8 miles per hour.
  - 1) According to Alice (in her reference frame), how fast is Bob moving and in what direction?
  - 2) According to Bob (in his reference frame), how fast is Alice moving and in what direction?

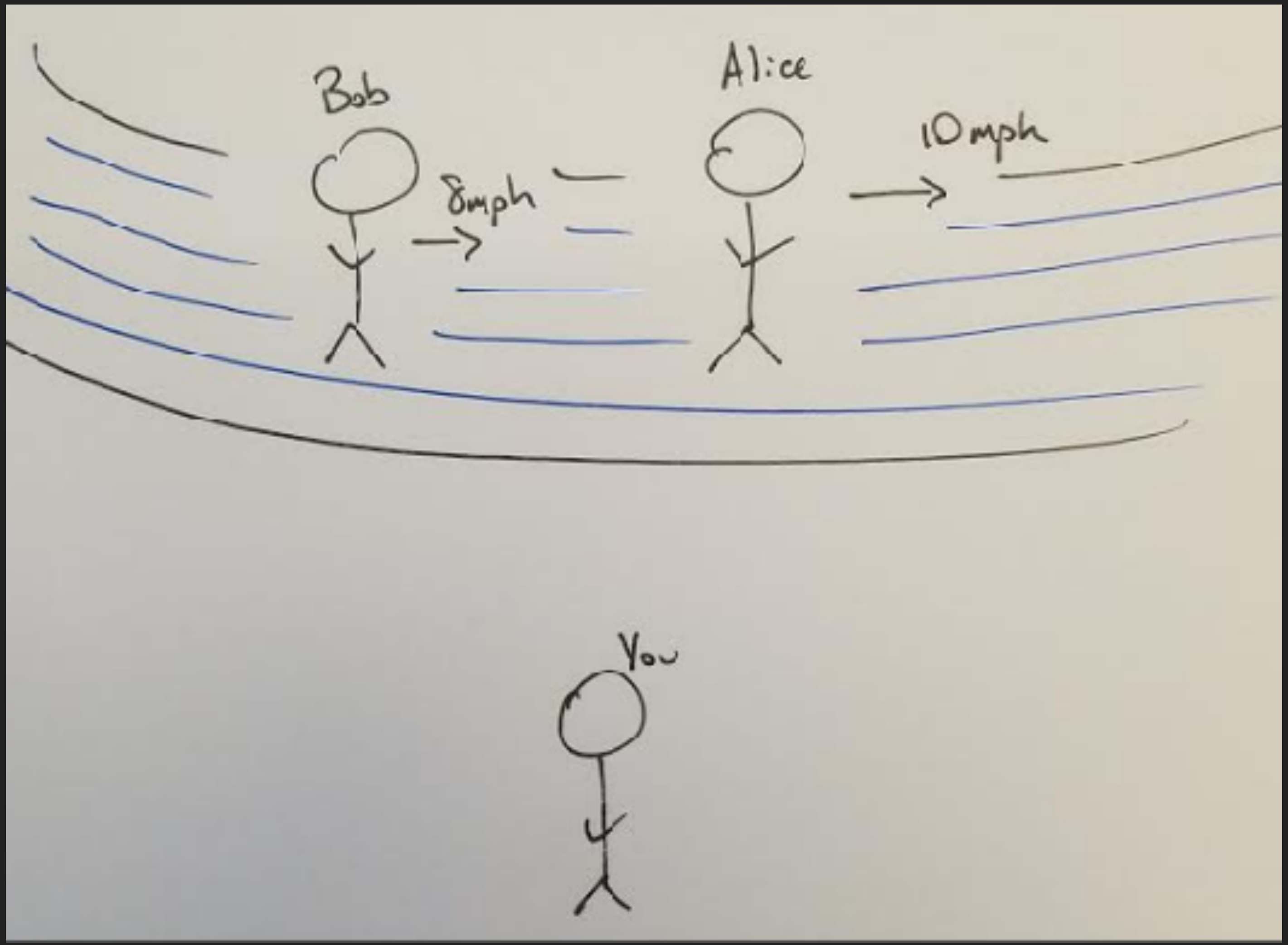
ANSWER OPTIONS:

A) 8 mph right, 10 mph right

B) 2 mph left, 2 mph right

C) 10 mph left, 8 mph right

D) 9 mph right, 9 mph right



Relative Velocities Part 2

- You are sitting in a row boat that is moving at 20 m/s with respect to the water, going North. You see a pirate ship coming towards you at a speed of 30 m/s.
  - Question 1) How fast is the pirate ship going with respect to the water?

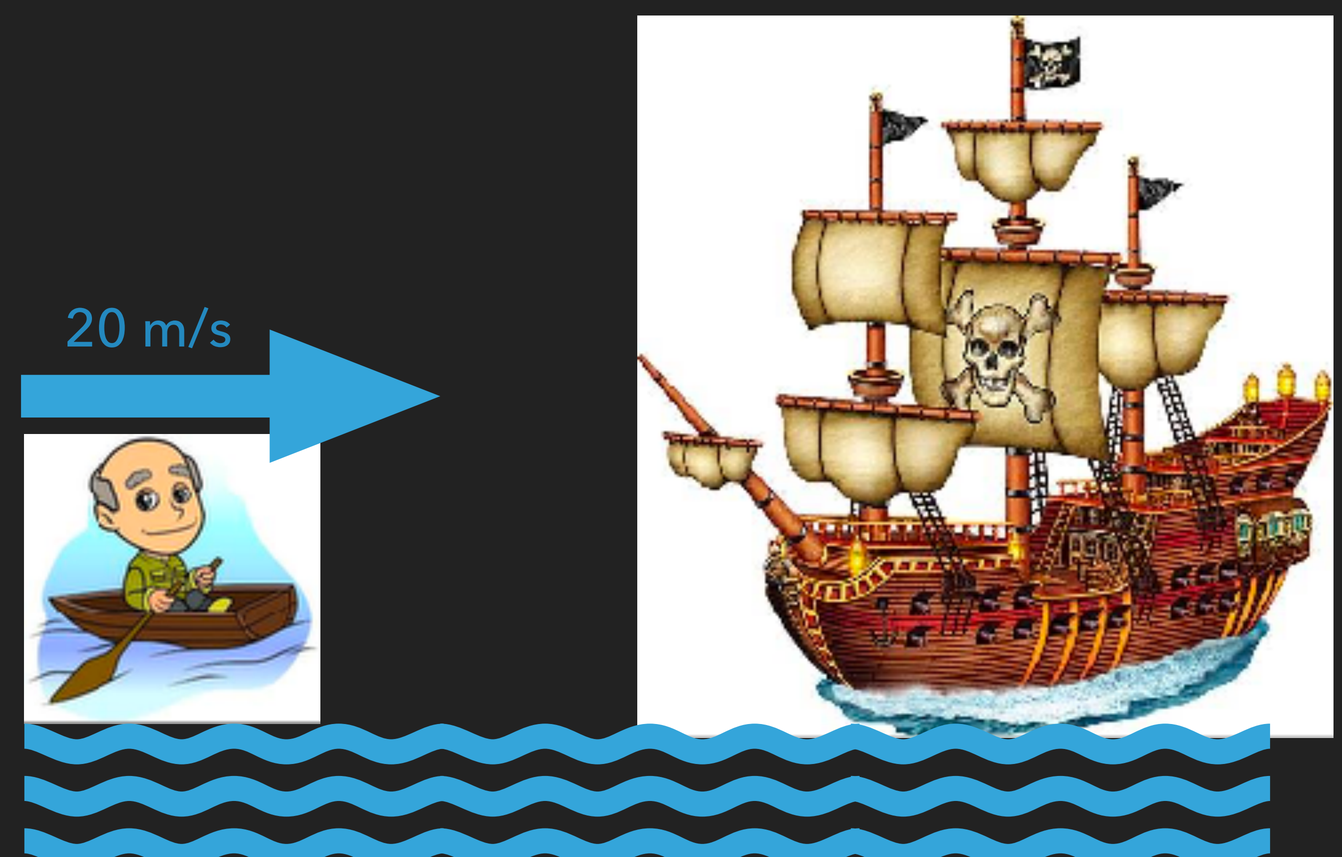
Question 1 OPTIONS:

A) 30 m/s

B) 20 m/s

C) 10 m/s

D) 0 m/s



Relative Velocities Part 2

- You are sitting in a row boat that is moving at 20 m/s with respect to the water, going North. You see a pirate ship coming towards you at a speed of 30 m/s.
  - Question 1) How fast is the pirate ship going with respect to the water?
- Suddenly, the pirate ship fires a cannonball that you see coming at your rowboat at a speed of 100 m/s.
  - Question 2) How fast must the pirates have fired the cannonball at you?

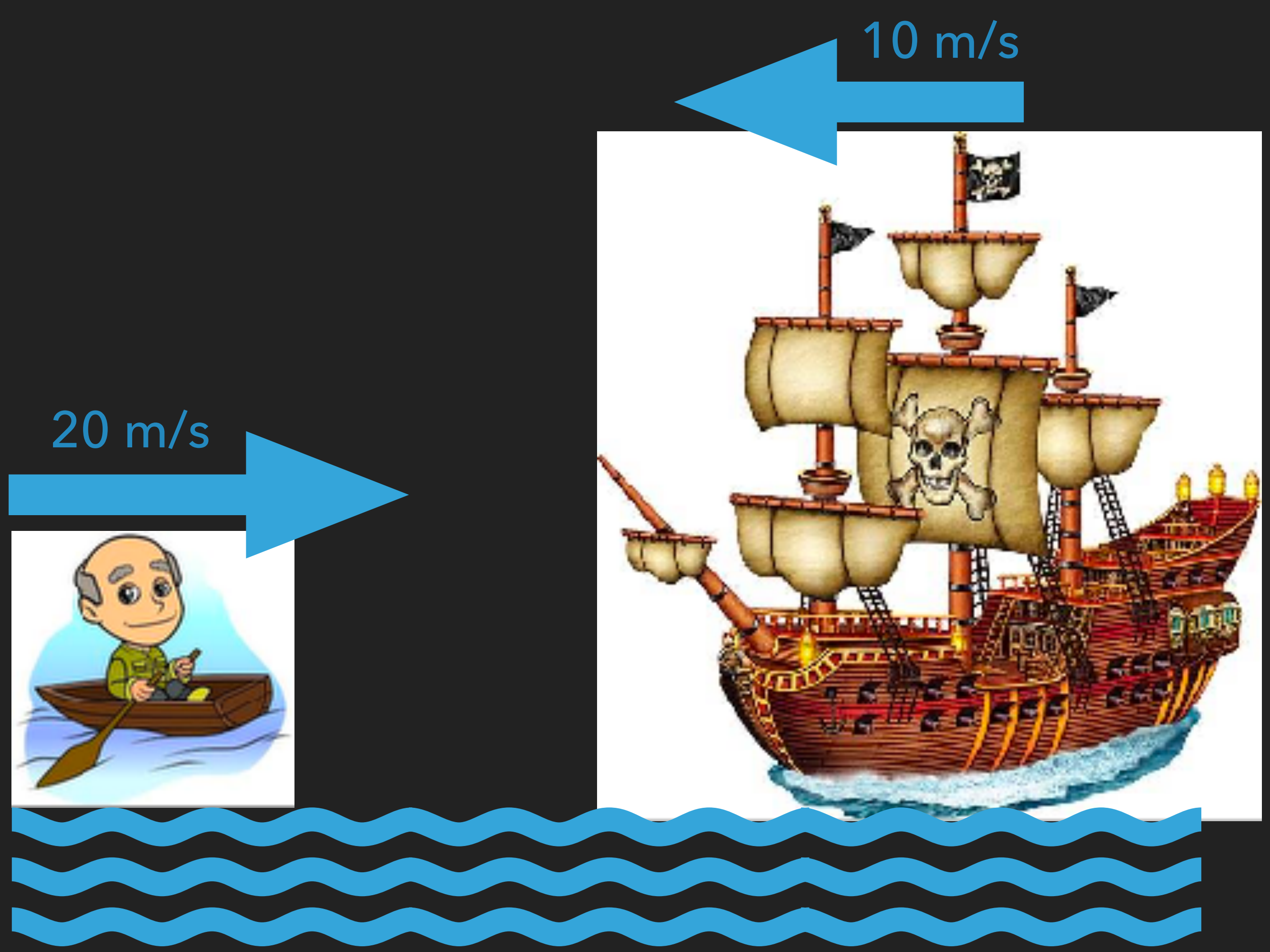
Question 2 OPTIONS:

A) 100 m/s

B) 80 m/s

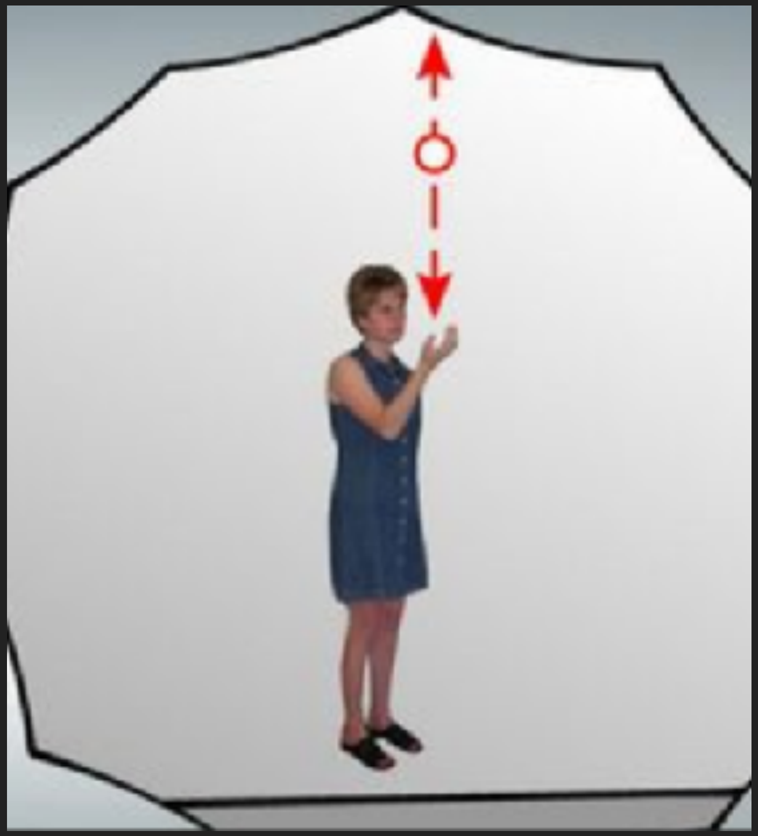
C) 70 m/s

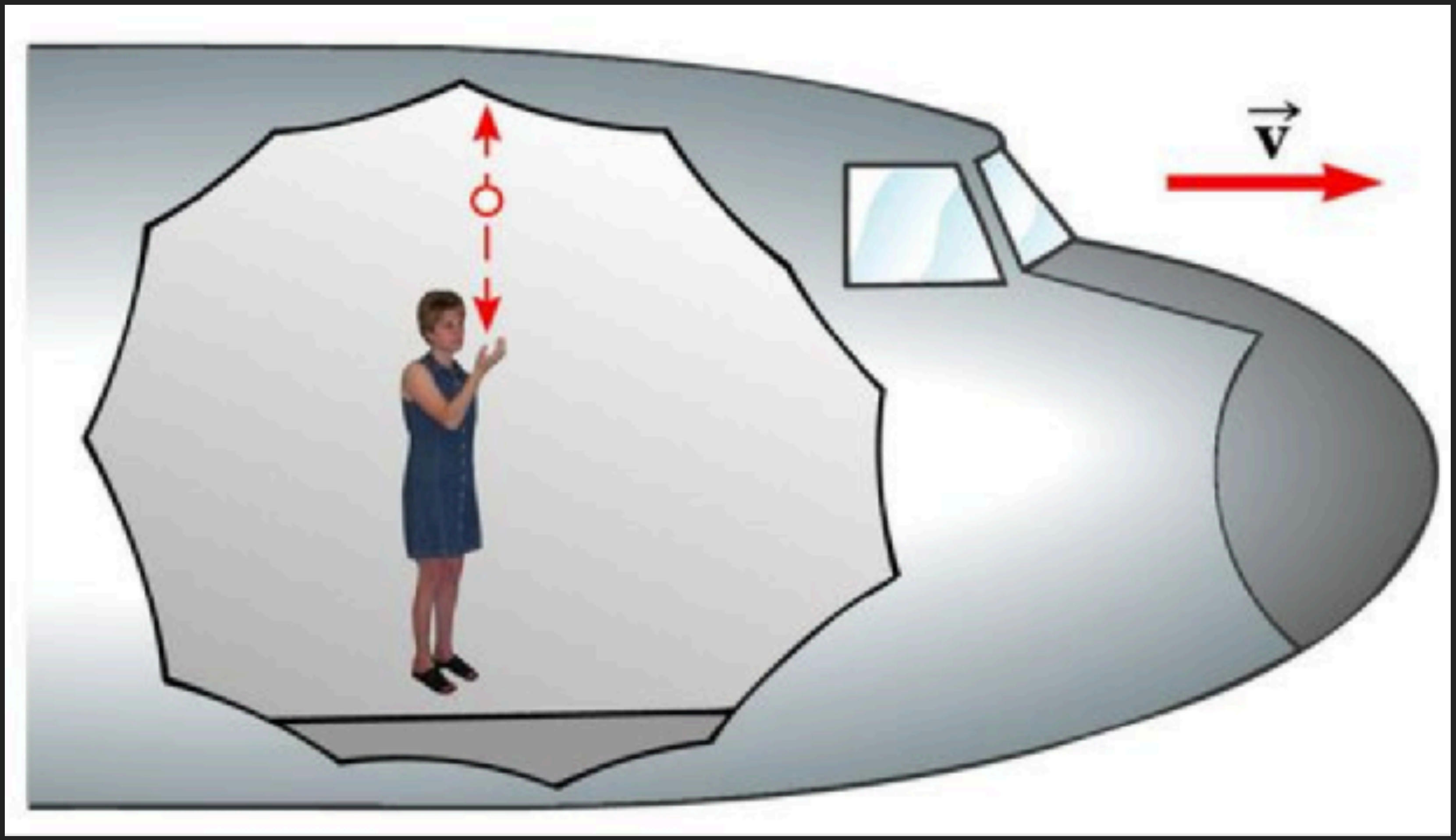
D) 50 m/s

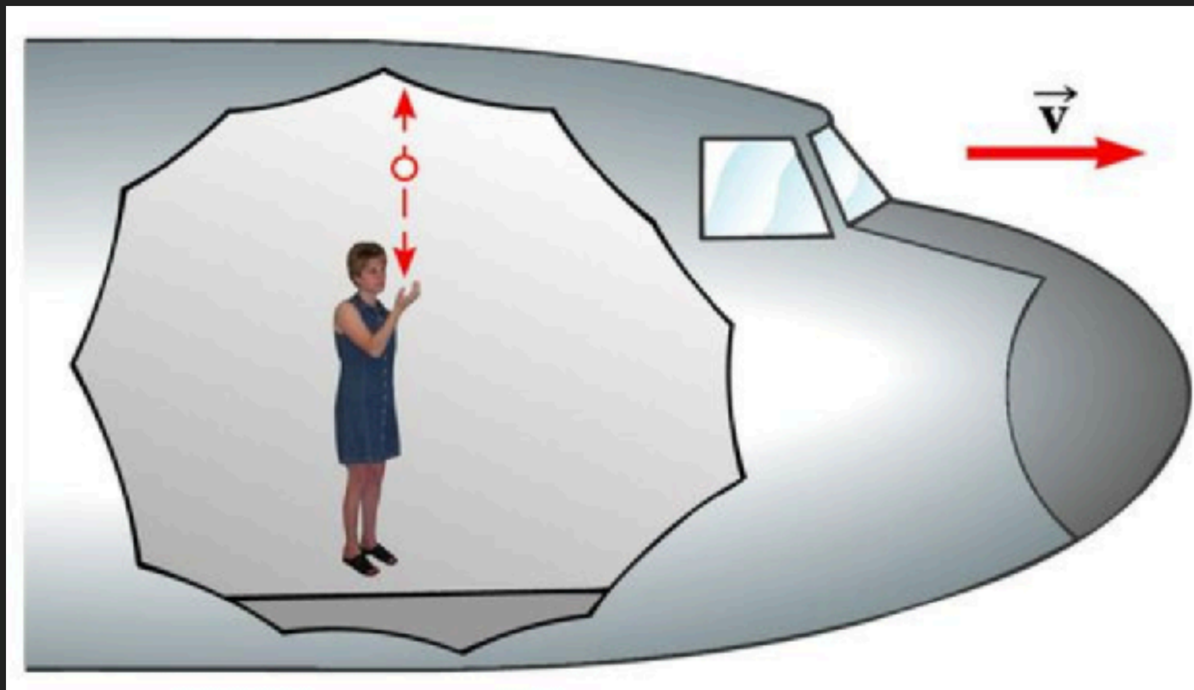


Before we get to Special Relativity, Galilean Relativity

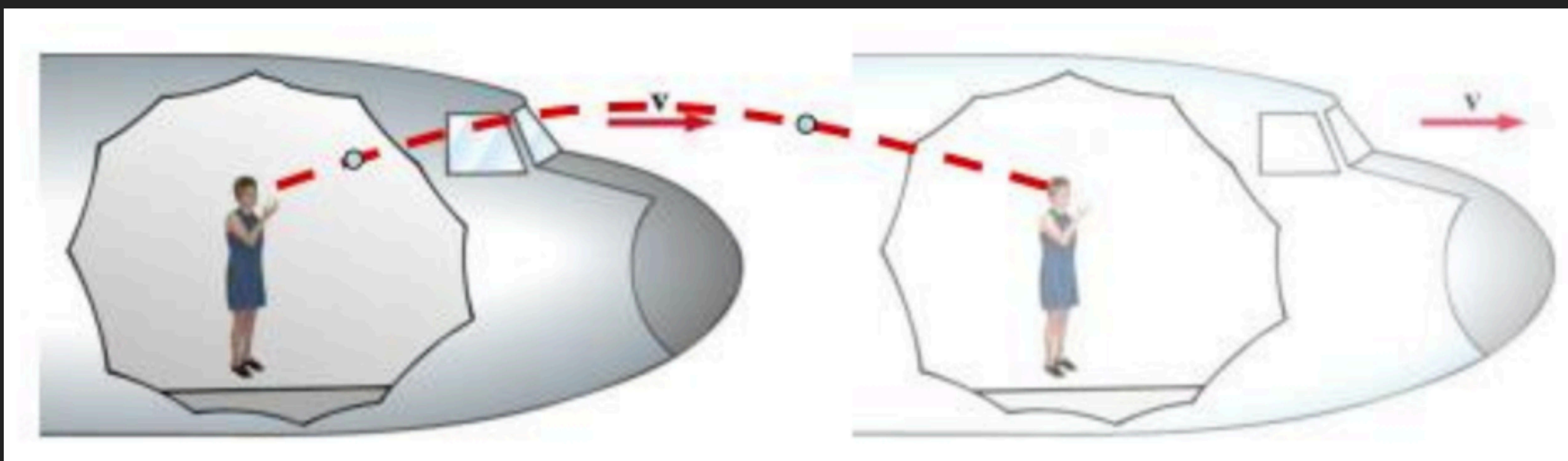
- Basic premise: what we see happening shouldn't depend on who watched it happen: someone moving vs. someone at rest, for instance.
- We know that the Earth is moving around the Sun, but we perform calculations for things as if Earth is at rest.







According to an observer on the plane, the ball goes up, then comes straight down

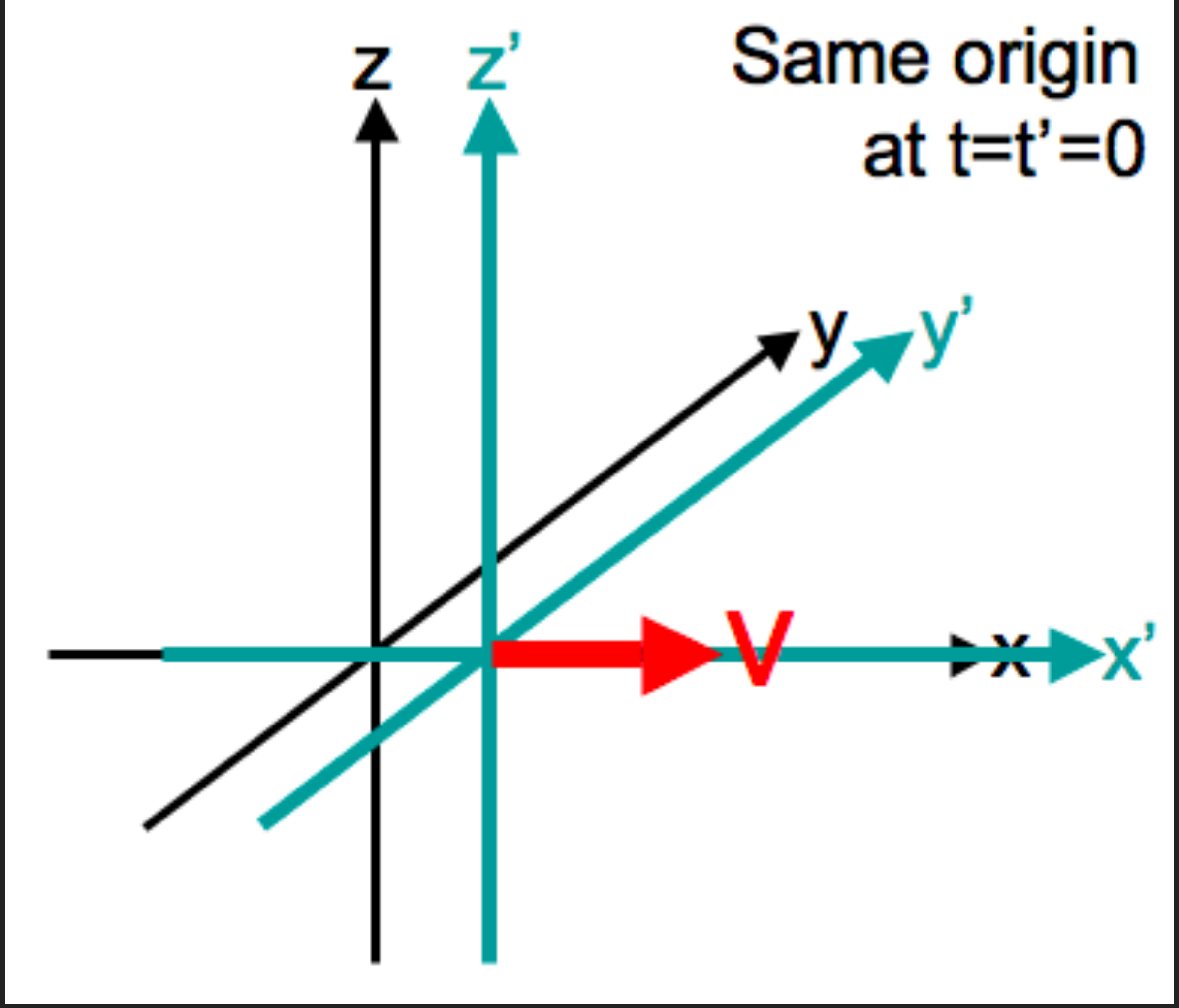


If observing from the ground though, the ball appears to fly up and to the right, then come down to the right and gets caught.

Because there is a relative velocity between the two observers, we must apply Galilean Relativity.

Galilean Transformations

One person's "reference frame" with respect to the other



$$x' = x - vt$$
$$t' = t.$$

Coordinates with apostrophes relate one observer's measurements (of position and time) to the other.

Newton's Laws and Galilean Transformations

Newton's laws of motion in physics	
LAW #1	A body at rest will remain at rest, and a body in motion will remain in motion unless it is acted upon by an external force.
LAW #2	The force acting on an object is equal to the mass of that object times its acceleration, F = ma.
LAW #3	For every action, there is an equal and opposite reaction.

Newton's Laws are "Invariant" under Galilean transformations – they give you the same prediction regardless of who measures the outcome (a moving observer vs. a stationary one, etc.)

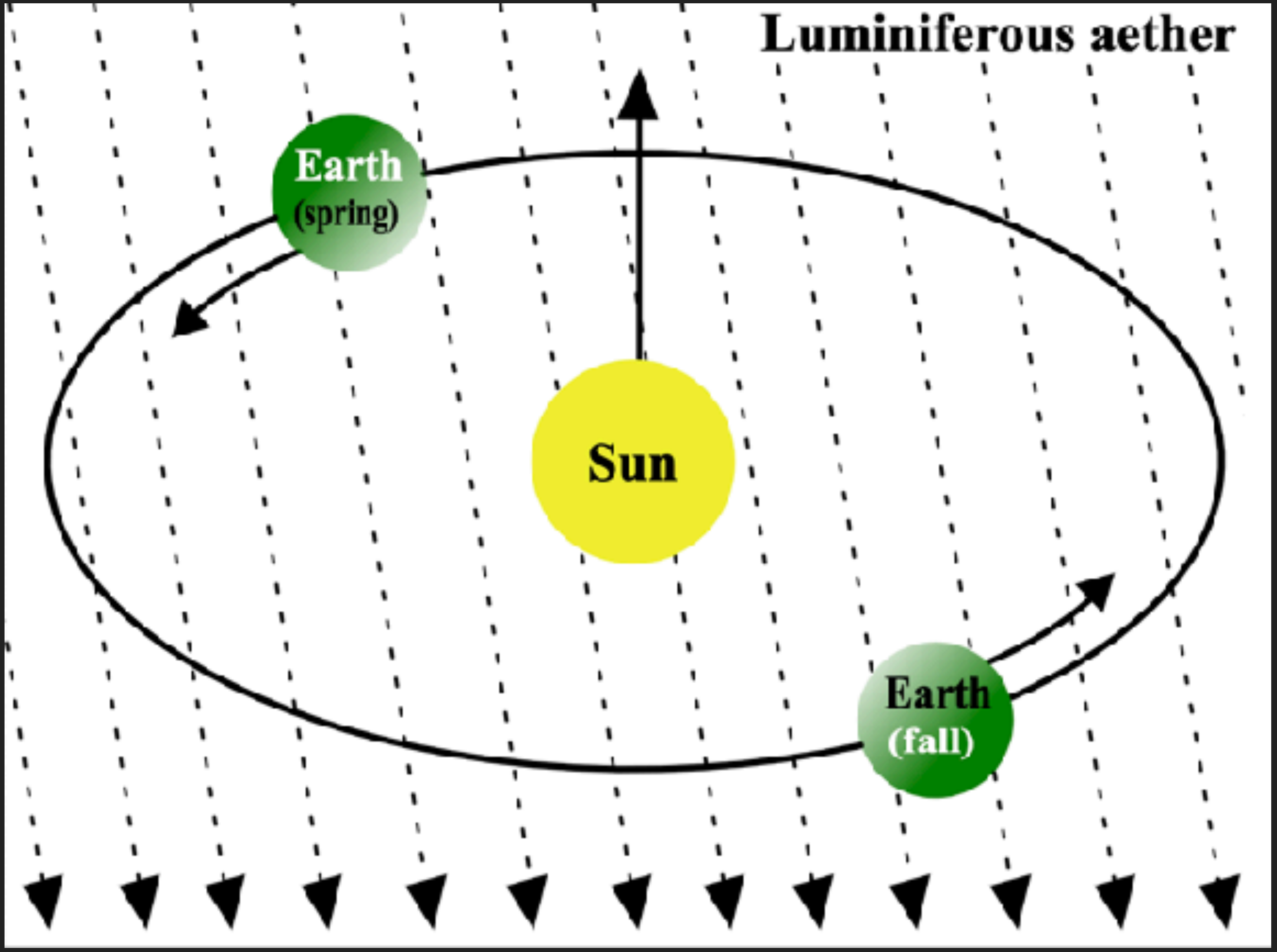
Back to Maxwell's Equations

$$\nabla \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}\right)$$

- The form of Maxwell's Equations imply that there is some "speed" that determines all of the relationships between electric and magnetic fields. We just saw that different observers can measure things to have different speeds.

- Is there some absolute rest frame in the universe that we're moving with respect to? Then we would measure the speed of light to be different based on which direction we're moving through that absolute rest frame…

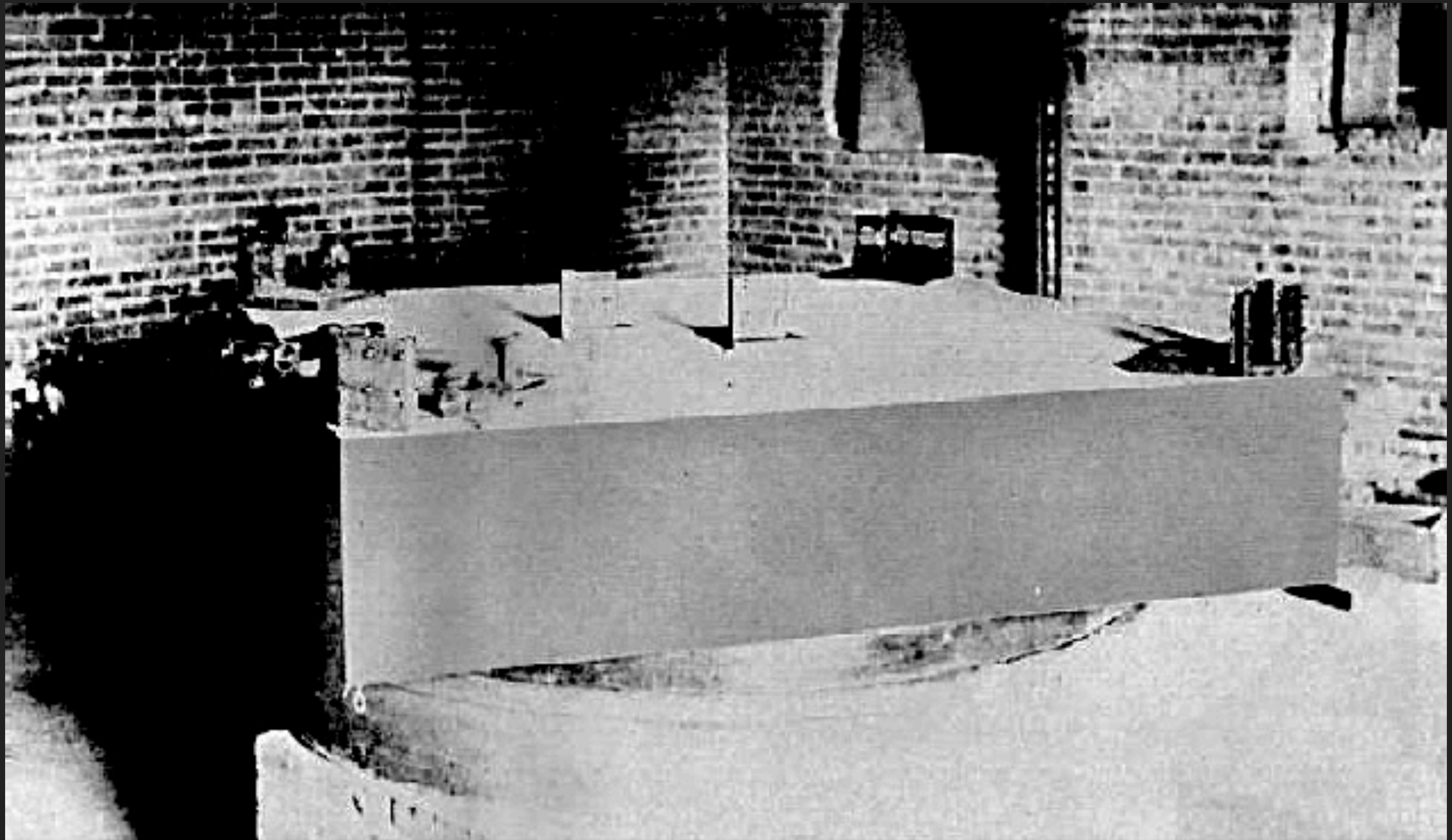
The “Luminiferous Aether”



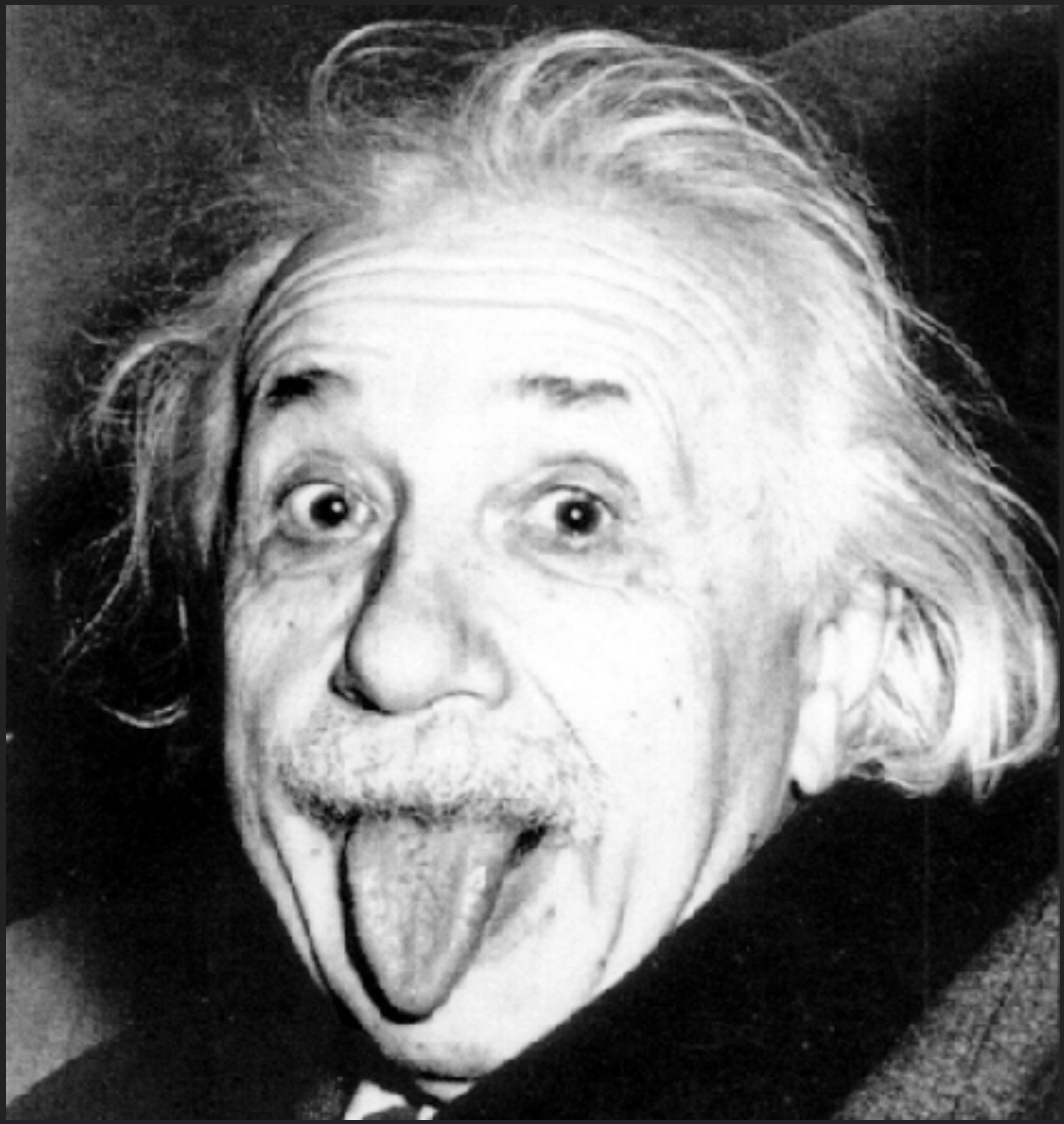
- Earth goes around the Sun, and the Sun moves around the Milky Way. If there is some “absolute rest frame” that allows light to propagate, we’re moving through it as we speak!
- Relatively speaking, we would move at different speeds through the aether depending on the season.

Michelson & Morley

- Albert Michelson & Edward Morley looked for this difference in light speed using a technique called "interferometry".
- They found no evidence of the aether!
- This seems to imply that no matter who measures it, light travels at the exact same speed!



Enter: Albert Einstein.



Einstein asked one (seemingly) simple question, and explored the consequences.

- What if
  - A) Physics does not depend on the reference frame of who is observing an event occurring?
  - And
  - B) The speed of light is constant in every reference frame?
- The consequences in a nutshell:
  - People will disagree on measurements of certain things (how long is this lecture going to last? Which event happened first?) but agree on others (the mass of a particle, etc.).

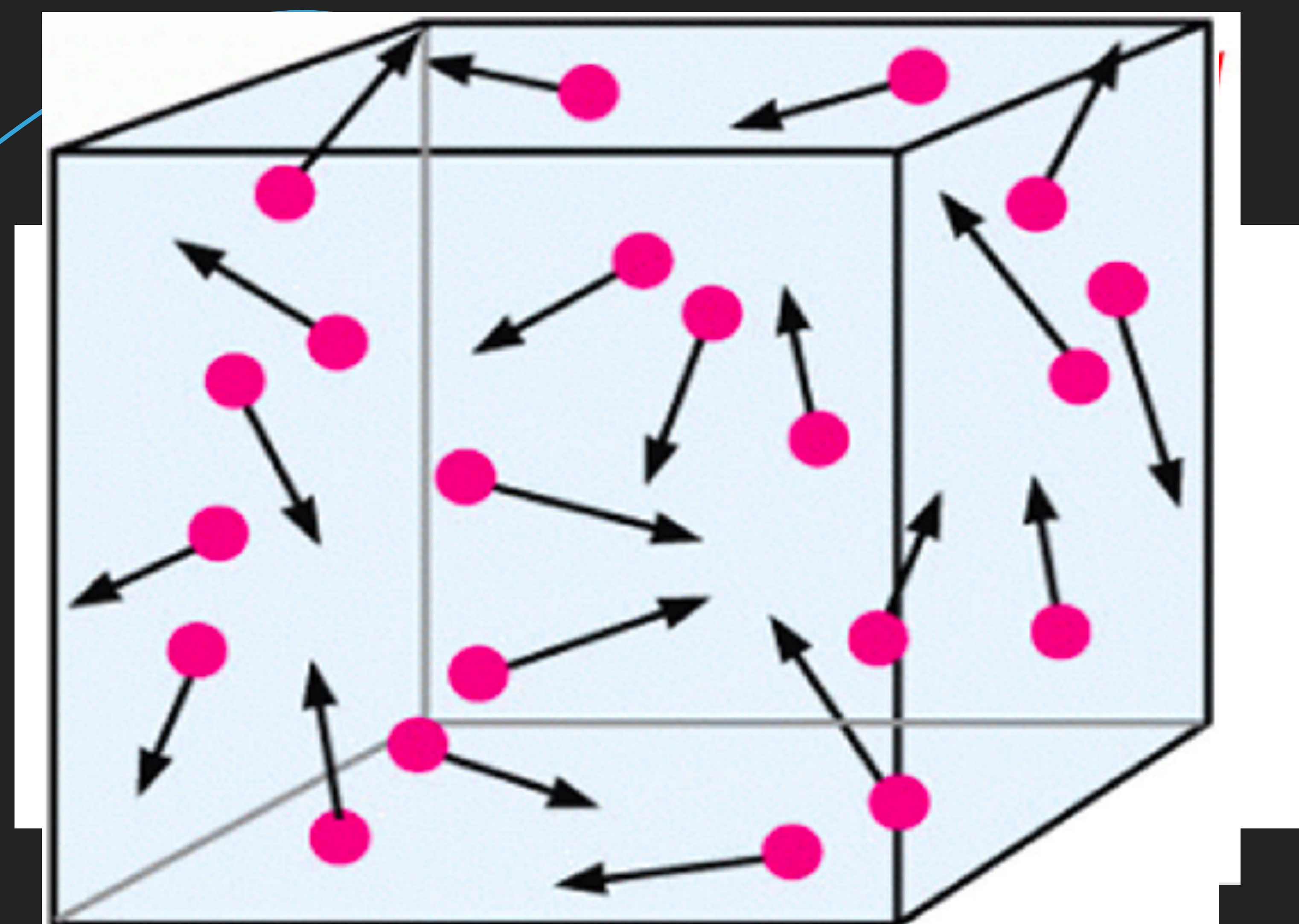
A revolution in Physics

- Einstein first published on this idea in 1905. That year has since been dubbed his “annubs mirabilis”, or “extraordinary year”.

- In 1905 alone, Einstein published four papers, each of which could arguably have deserved a Nobel prize:

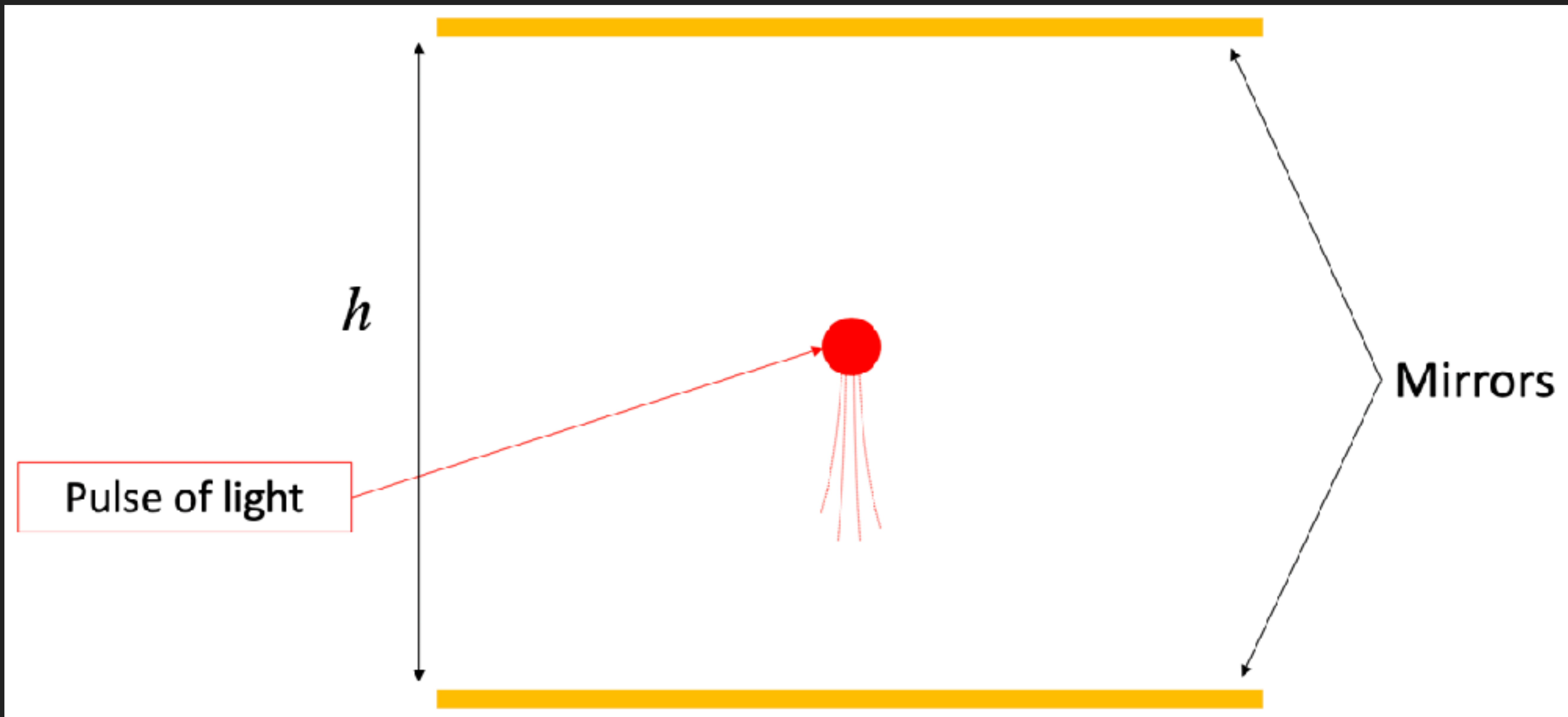

The photoelectric effect

- Brownian Motion
- Special Relativity
- Energy-Mass Equivalence



We're about to use a bit of algebra to explore the consequences of Einstein's question. Please, if I am going too quickly, stop me and ask questions.

Let's Explore the Consequences: Time Dilation



Imagine we have a "light clock" that ticks by shooting a pulse of light from one mirror to another. The mirrors are separated by a distance "h", and the light pulse travels at a speed "c".

$$\text{distance} = \text{rate} \times \text{time} \longrightarrow d = rt$$

$$h = ct \implies t = \frac{h}{c}$$

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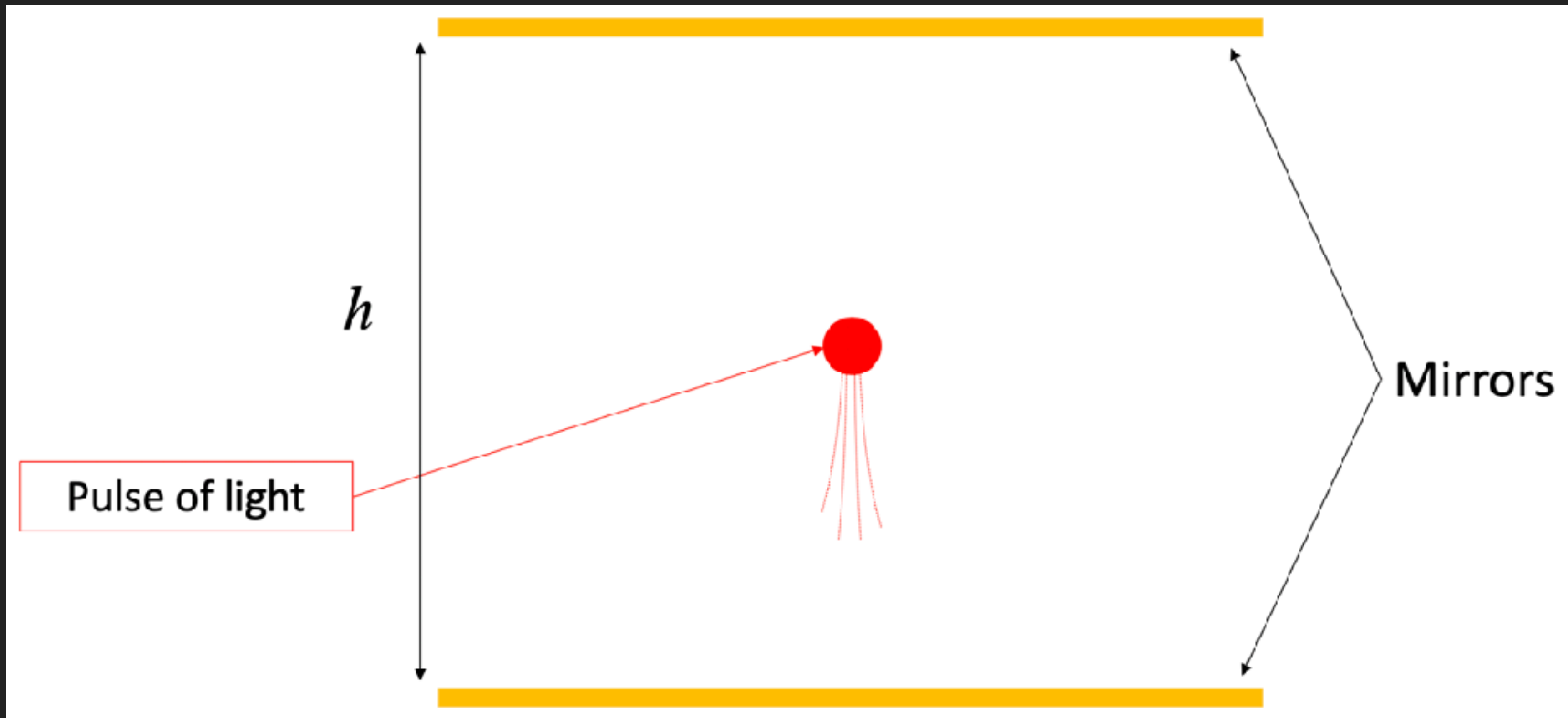


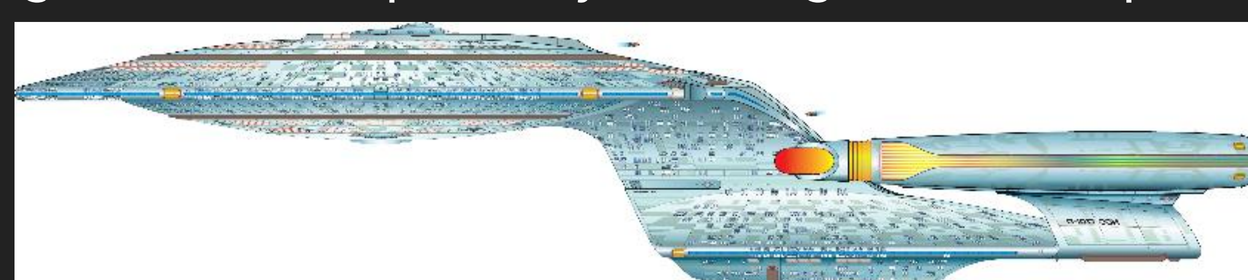
$$h = ct \implies t = \frac{h}{c}$$

If we want this clock to tick once per second, we have to put these mirrors pretty far away (one light-second):

$$h = ct = (3 \times 10^8 \text{ m/s})(1 \text{ s}) = 300{,}000{,}000 \text{ m}$$

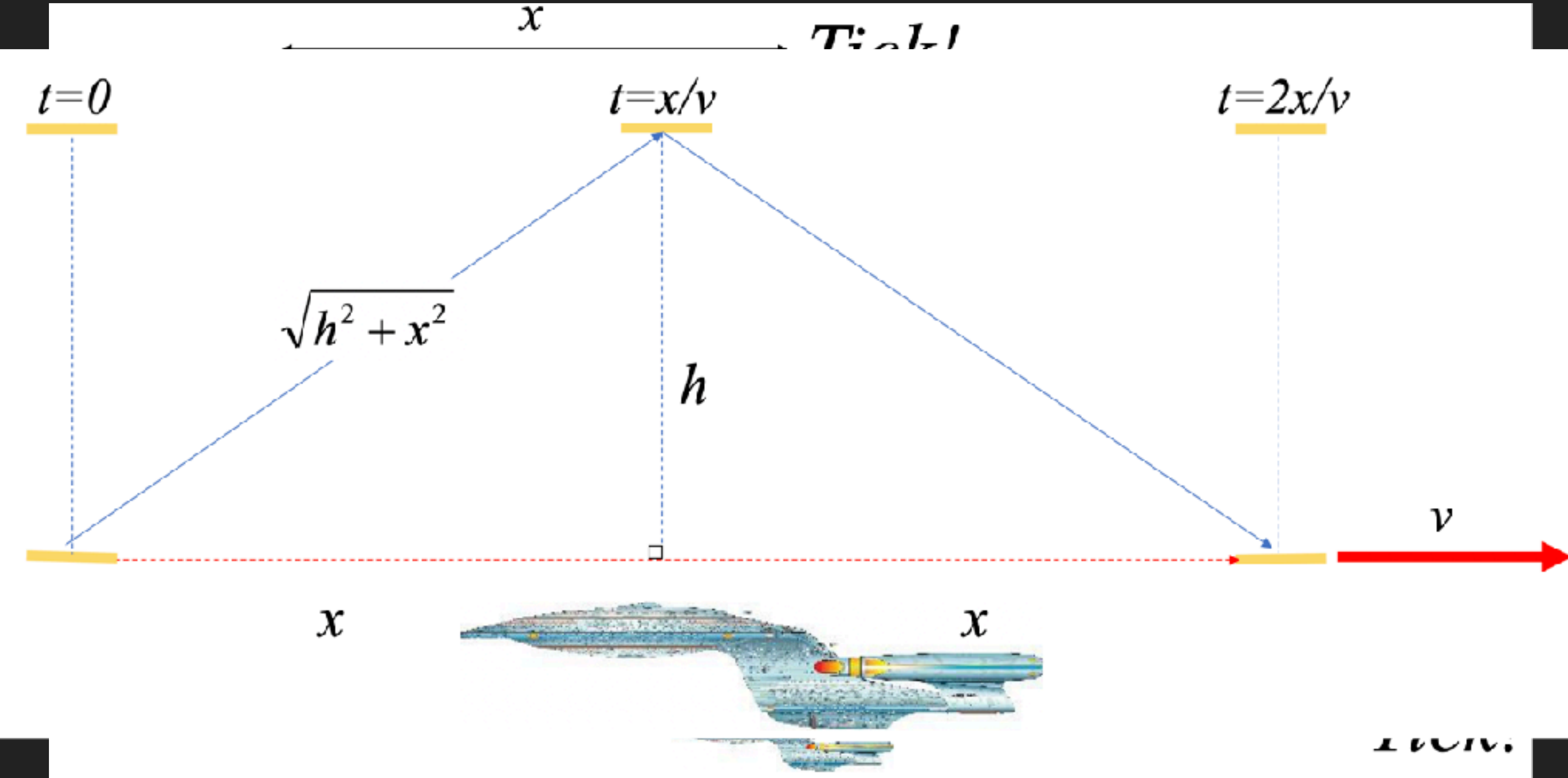
This is *almost* the distance between the Earth and the Moon!

This was all assuming we're sitting at rest and the mirrors are also at rest. Now, let's imagine we have a friend flying his rocket ship nearby as the light clock is operating.



Your friend's ship is the USS Enterprise, obviously!

As viewed by the spaceship,

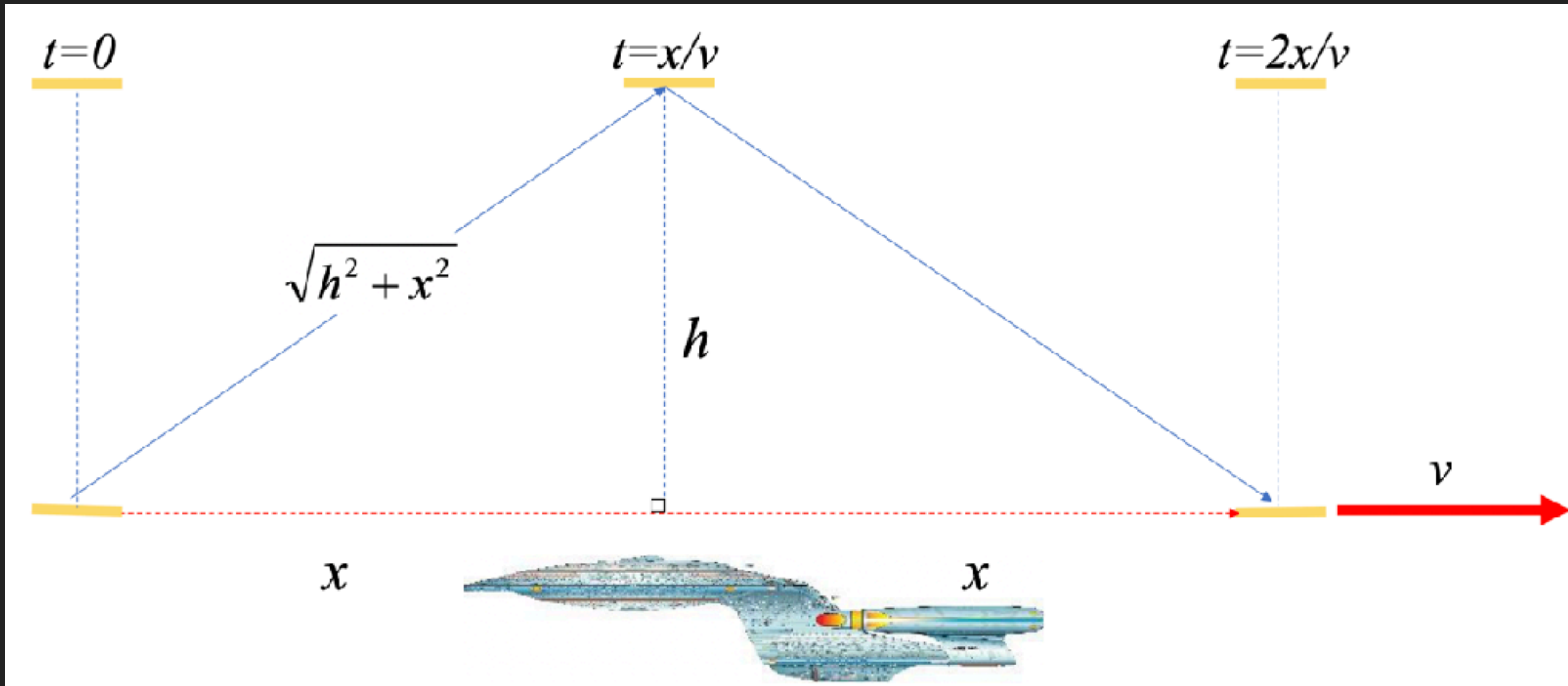


Instead of traveling a distance “h” between ticks, the light pulse actually travelled further.

$$h \implies \sqrt{h^2+x^2} = \sqrt{h^2+(vt)^2}$$

$$\text{Pythagorean Theorem}: \quad a^2+b^2=c^2, \quad c=\sqrt{a^2+b^2}$$

As viewed by the spaceship,



$$d = \sqrt{h^2 + (vt)^2}$$

$$t = \frac{d}{c} = \frac{\sqrt{h^2 + (vt)^2}}{c}$$

We have an equation that has "t", the time as measured by the Enterprise pilot, on both sides. Let's solve for "t".

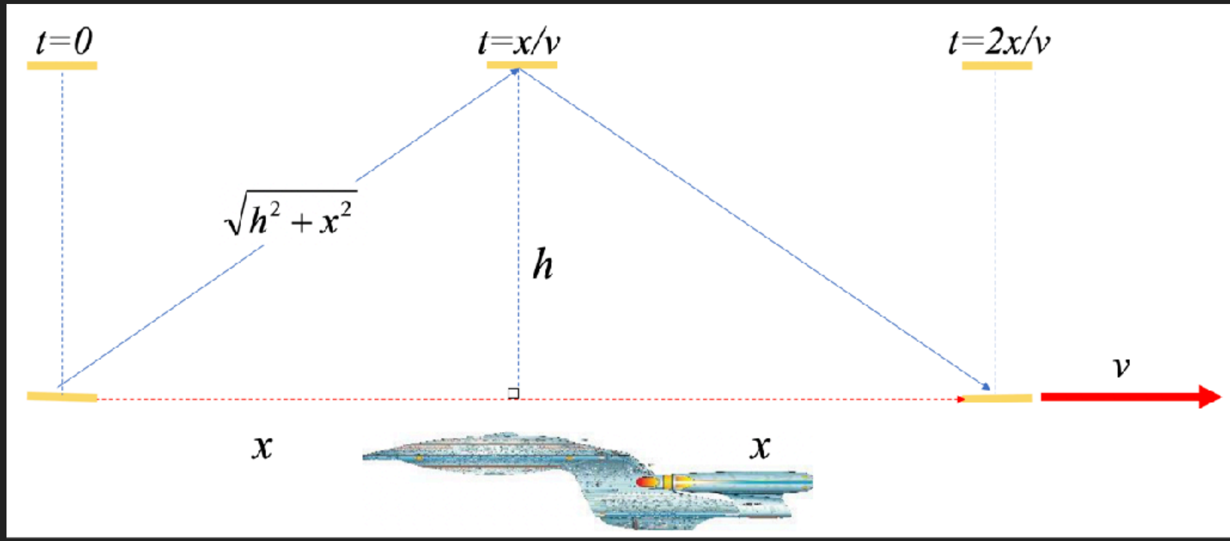
$$ct = \sqrt{h^2 + (vt)^2}$$

$$c^2t^2 = h^2 + v^2t^2$$

$$c^2t^2 - v^2t^2 = h^2$$

$$\left(c^2 - v^2\right) t^2 = h^2$$

"t" as measured by the spaceship



$$\left(c^2 - v^2\right) t^2 = h^2$$

$$c^2\left(1 - \frac{v^2}{c^2}\right) t^2 = h^2$$

$$\beta \equiv \frac{v}{c}, \quad (0 \leq \beta < 1)$$

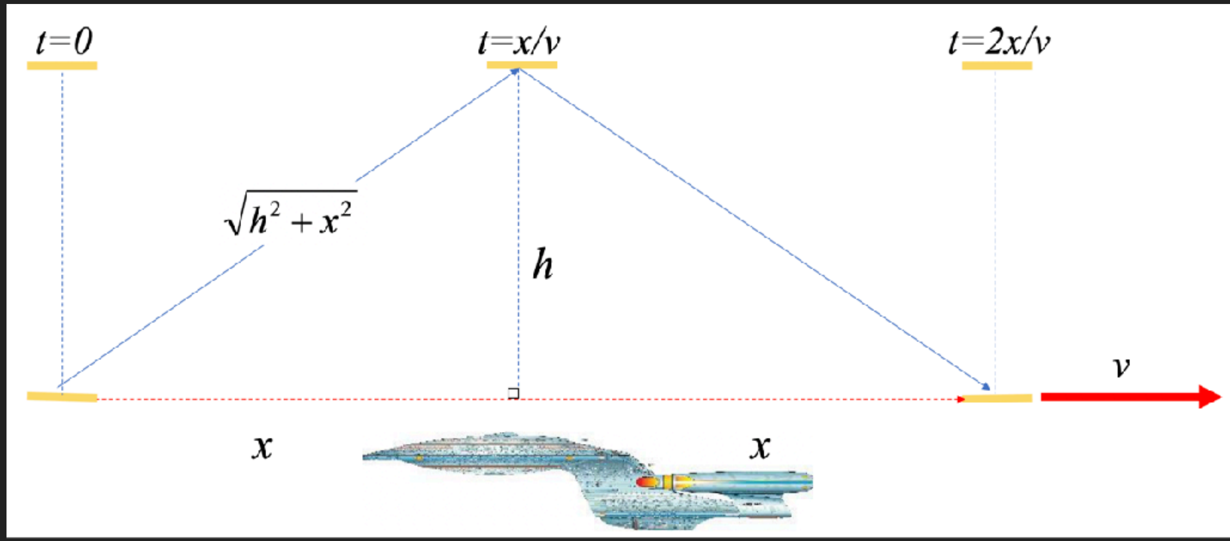
$$c^2\left(1 - \beta^2\right) t^2 = h^2$$

$$t^2 = \frac{h^2}{c^2} \frac{1}{\left(1 - \beta^2\right)}$$

$$\boxed{t = \frac{h}{c} \frac{1}{\sqrt{1 - \beta^2}}}$$

Remember, according to the stationary observer, $t = \dfrac{h}{c}$

Time Dilation



$$t' \equiv t_{\text{moving}} = \frac{h}{c}\frac{1}{\sqrt{1-\beta^2}} \quad \text{Vs} \quad t \equiv t_{\text{still}} = \frac{h}{c}$$

According to the moving observer, the clock's time has been "dilated" by a factor of

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \implies t' = \gamma \times t$$

Gamma

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- Is time always "dilated"? What's the relationship between

$$t' \text{ and } t?$$

$$\beta \equiv \frac{v}{c}, \quad (0 \leq \beta < 1)$$

Talk to your neighbor: what happens as

$$\beta \to 0?$$

$$\beta \to 1?$$

Second Consequence: Length Contraction

- How long is the spaceship according to someone standing on the spaceship's deck? L
- How long is the spaceship according to someone standing and watching it fly by? L'

(Remember: both the spaceship pilot and the person watching agree that the other is moving at a velocity v)

According to the spaceship pilot, it covers this distance in a time t: $L = vt_{\text{Ship}}$

Now, the observer watching this happen is moving, and measures it in a time t': $L' = vt_{\text{Clock}}$

According to the observer watching, the pilot's ship will run faster, by $t_{\text{Ship}} = \gamma t_{\text{Clock}}$

$$L = vt_{\text{Ship}}$$

$$L' = vt_{\text{Clock}} = vt_{\text{Ship}}\frac{1}{\gamma}$$

$$L' = \frac{L}{\gamma}$$

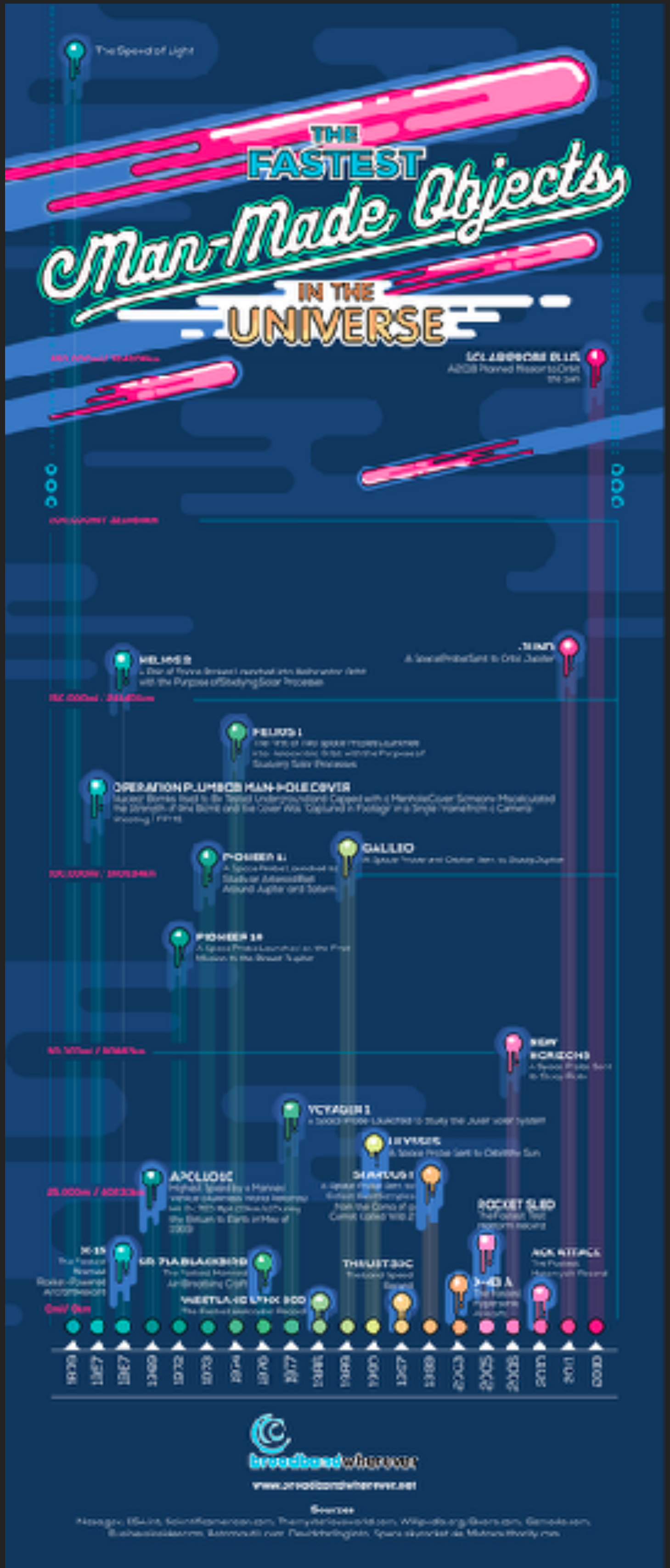
An observer at rest thinks a length is longer than an observer moving with respect to it!

So, why don’t we notice these weird effects?

$$\boxed{t = \frac{h}{c}\frac{1}{\sqrt{1-\beta^2}}}$$

$$\boxed{d' = \frac{d}{\gamma}}$$

Recall, the speed of light is quite large, roughly 300,000,000 meters per second.

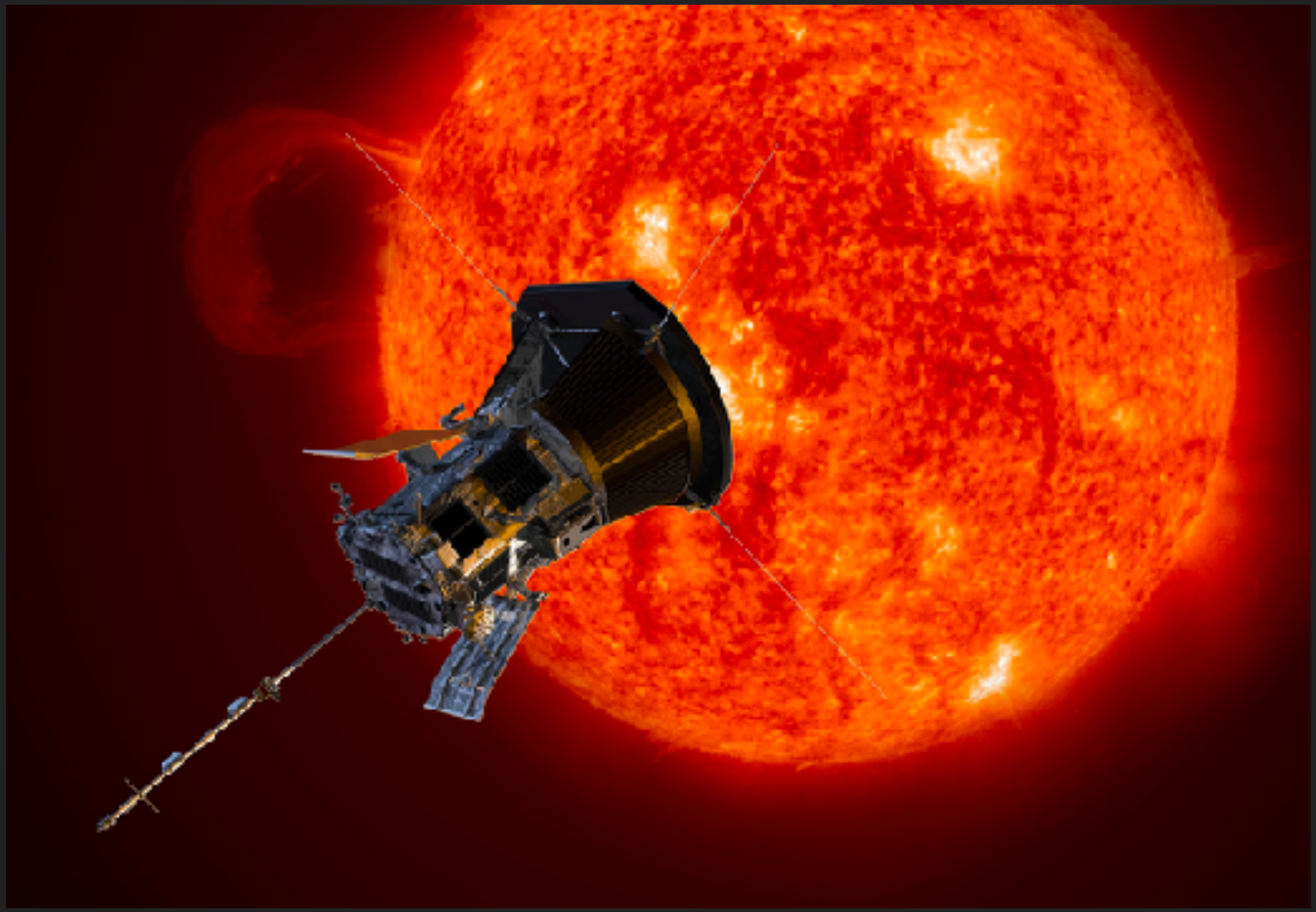


So, why don't we notice these weird effects?

Recall, the speed of light is quite large, roughly 300,000,000 meters per second.

$$\boxed{t' = \gamma t = t\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$\boxed{L' = \frac{L}{\gamma}}$$



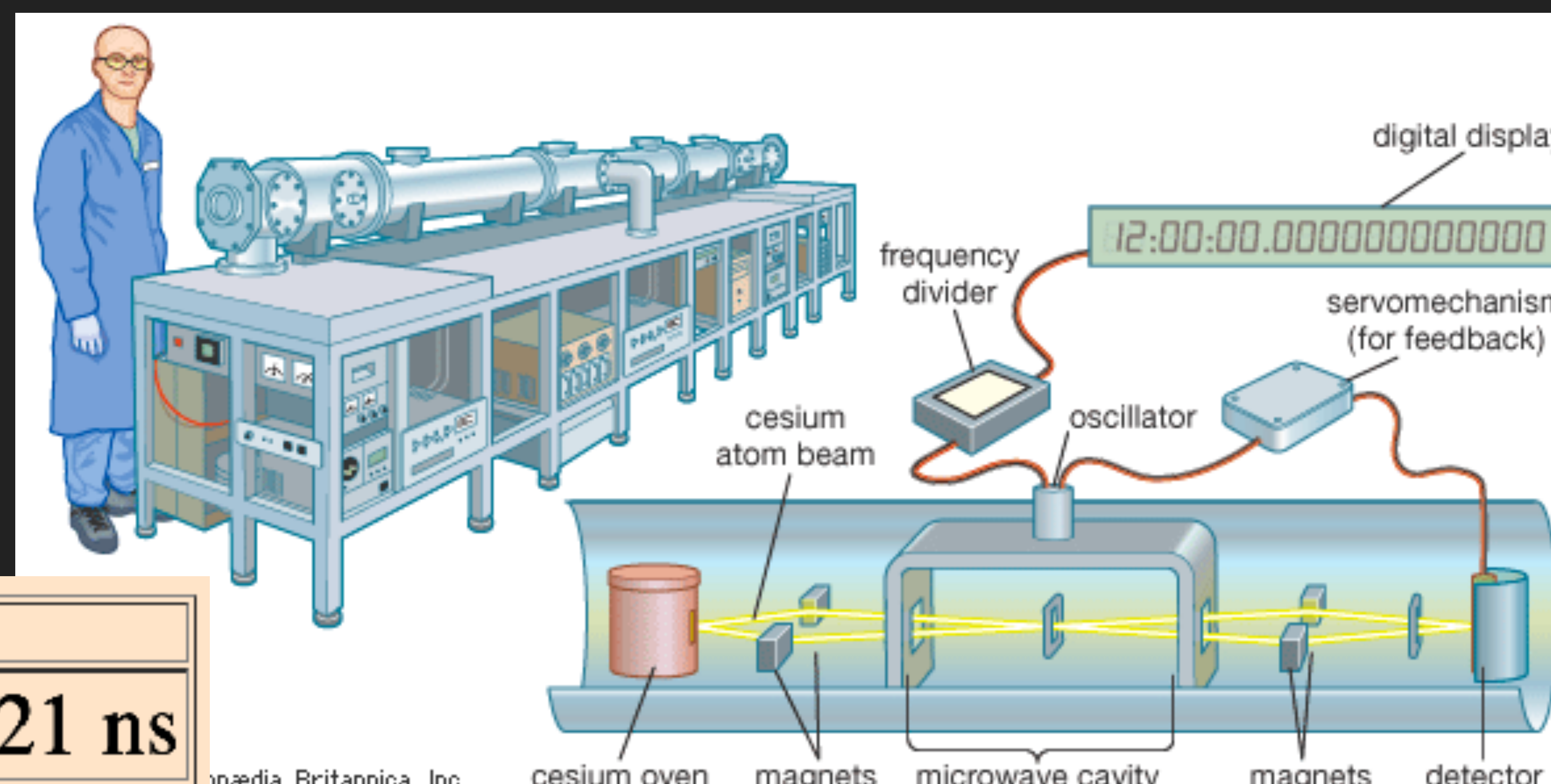
The Parker Solar Probe, currently en route to the sun. Maximum planned velocity: 430,000 miles per hour. In the same units, that's almost 200,000 meters per second.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(200000\ \mathrm{m/s})^2}{(300000000\ \mathrm{m/s})^2}}} = 1.00000022$$

Let's assume the PSP is 10 m long. This says that a moving observer would measure it to be about 2 microns (the size of a bacteria!) shorter than it actually is. Good luck measuring that!

Actually, we have measured these effects!

- It may be hard to measure tiny changes to a length (contraction), but tiny changes to a time (dilation) can build up over time.
- Atomic clocks are extremely precise and can measure down to nanosecond-level precision.
- In 1971, cesium atomic clocks were flown around the world, one on an east-bound jet and one on a west-bound jet. Relative to cesium clocks that stayed at the US Naval Observatory, the two that flew around the world lost time!



	Eastward Journey	Westward Journey
Predicted	-40 +/- 23 ns	+ 275 +/- 21 ns
Measured	-59 +/- 10 ns	+ 273 +/- 7 ns

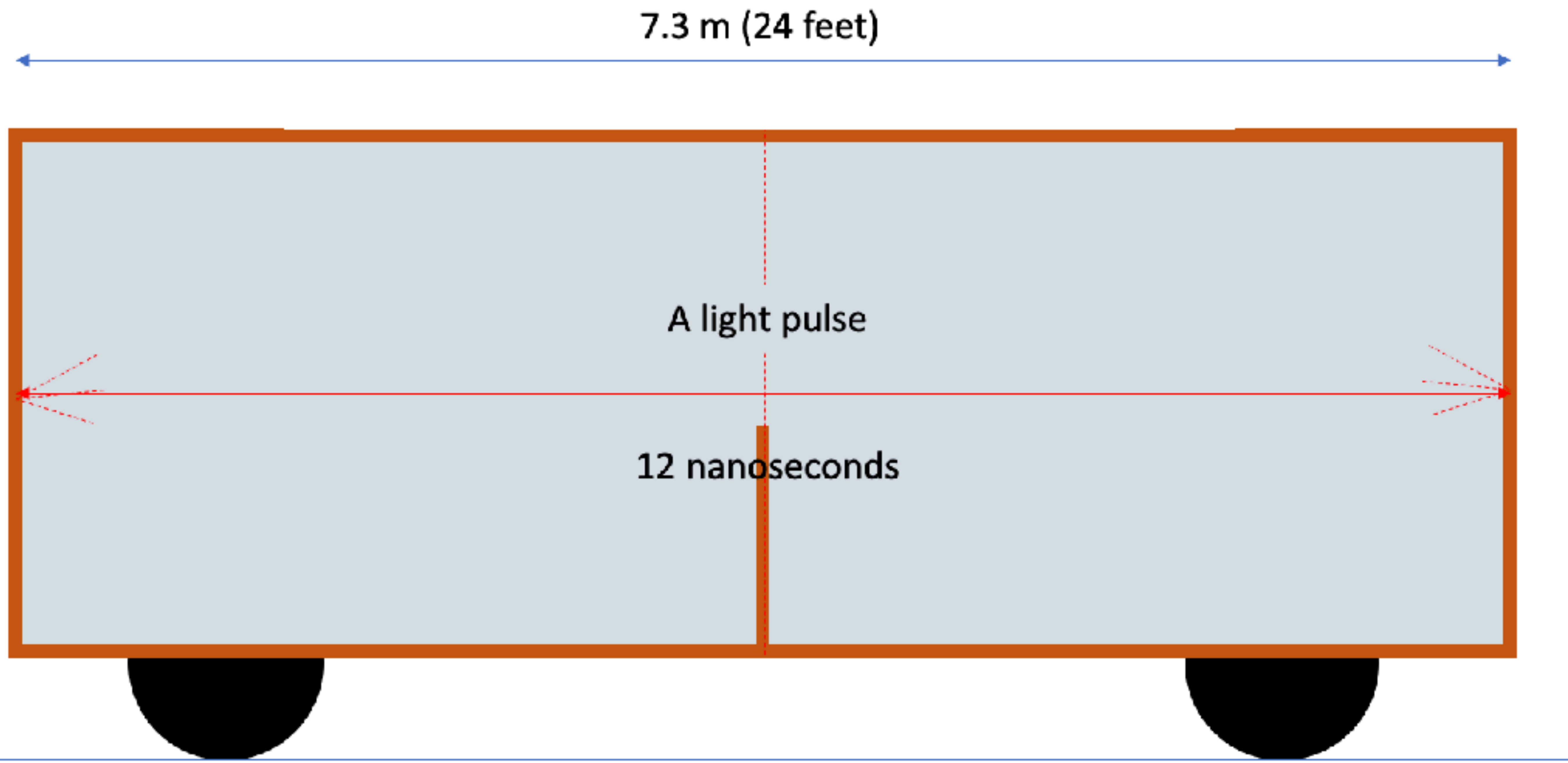
Note: there's more to this prediction than just special relativity. We'll get to that in a bit!

Last Consequence: Simultaneity

- Simultaneity: a fancy way of saying things happen "simultaneously", or at the same time.

Thought-experiment: The Train Car

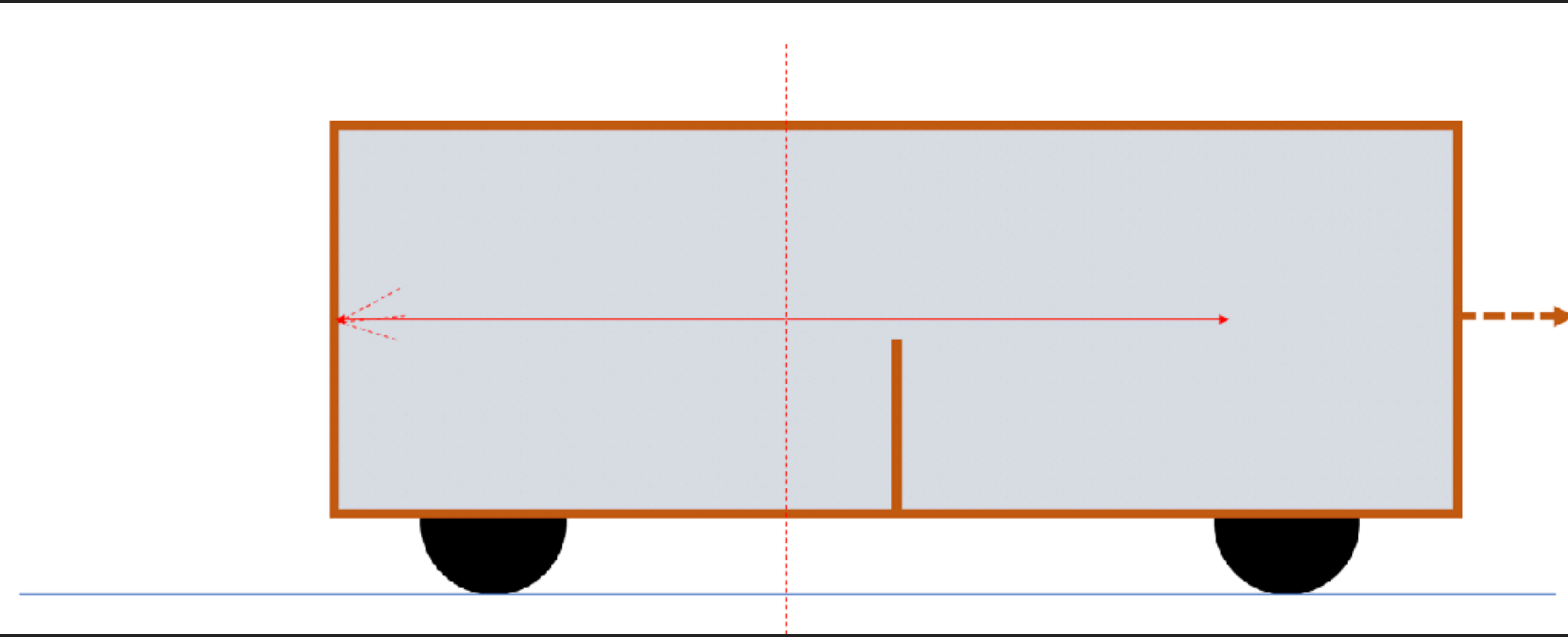
Imagine you're sitting on a train car that's about 24 feet long. At time t = 0 s, a light pulse is emitted from the center of the car.



As you watch it travel through the car, you see that it takes 12 ns to reach each edge of the train car. You also see the pulse hit both sides of the car at the same time. Nothing too wild, right?

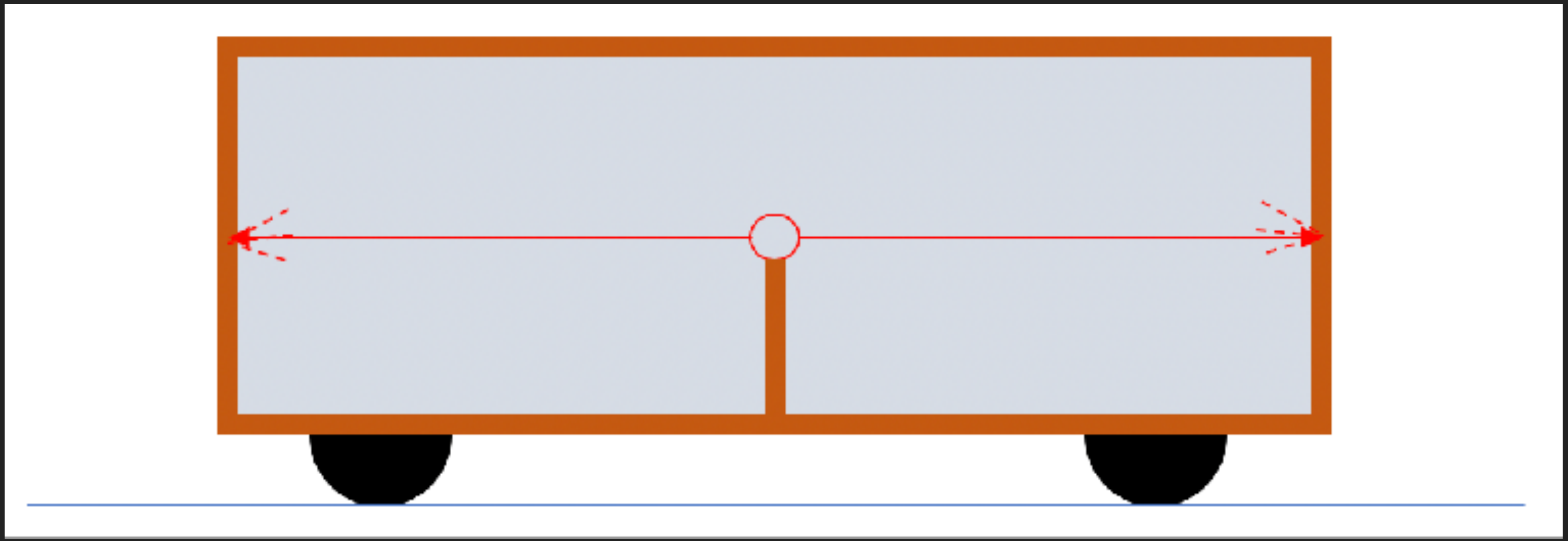
Rethinking the Train Car

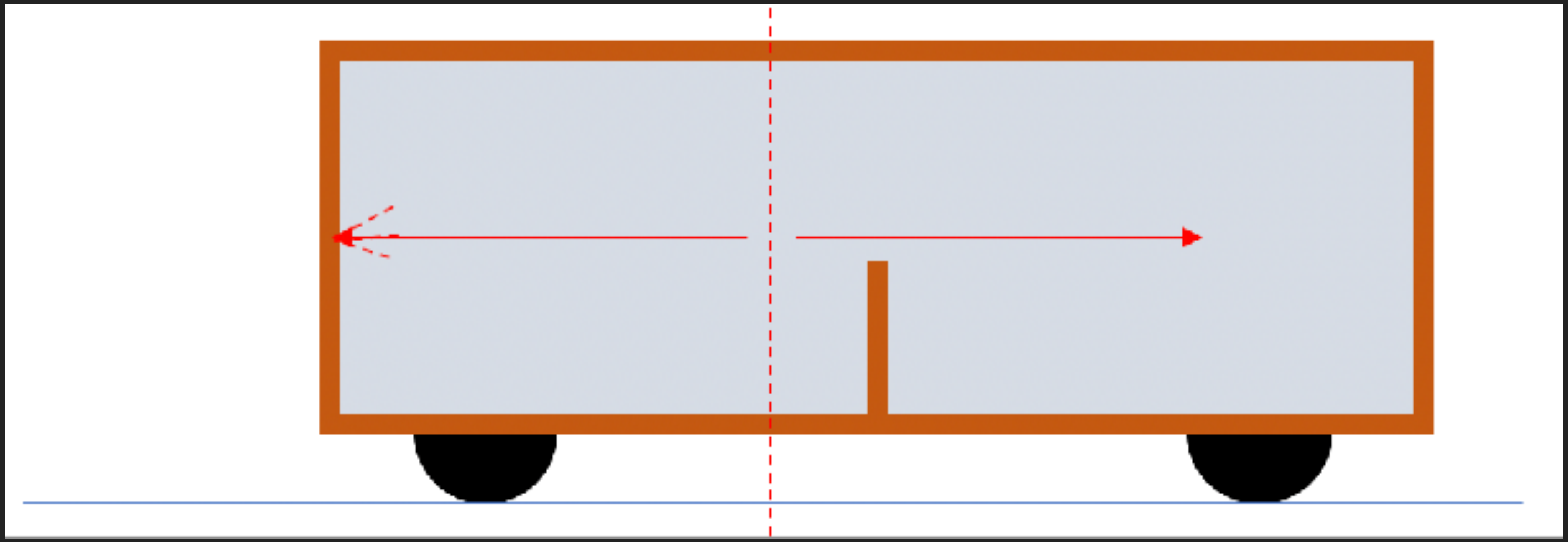
Now, imagine you're watching the train car roll by as you sit outside. As the center of the car reaches your vantage point, the same pulse gets emitted.



From the outside, the light pulse hits the back of the car before it hits the front of the car!

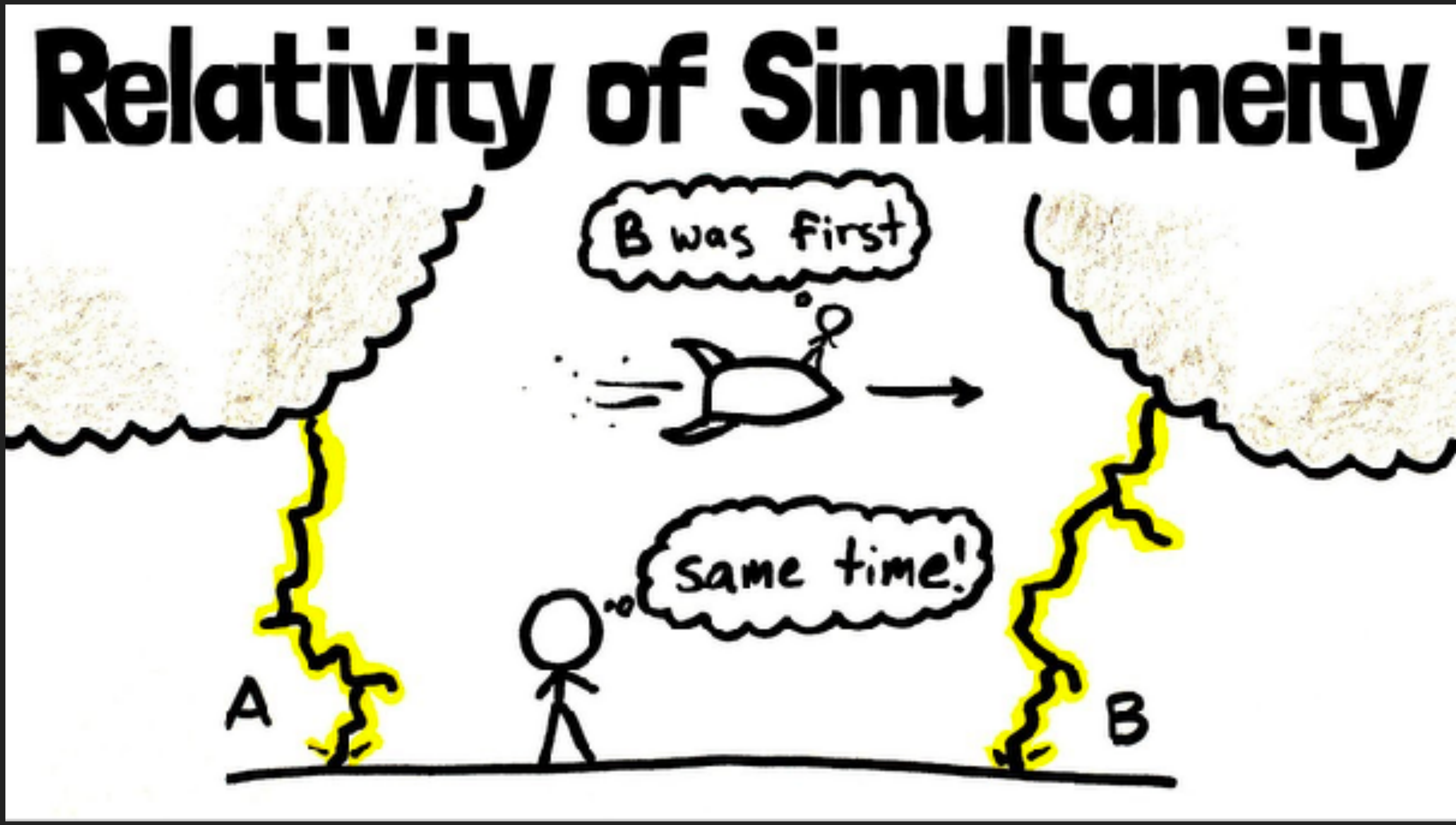
Different Observers Disagree about the Order of Events!





Causality

- So, two observers can disagree about the order of when things happened. Can they disagree about whether one event impacted another?



https://www.youtube.com/watch?v=SrNVsfkGW-0

Summary of Consequences:

- Time Dilation:
  - Moving observers measure the passage of time to be shorter than stationary observers.
- Length Contraction:
  - Moving observers measure objects to have a shorter length than an observer at rest does.
- Simultaneity:
  - Moving observers and stationary observers may disagree on the order of events occurring.

Weird Paradoxes

The Pole Vaulter and the Barn

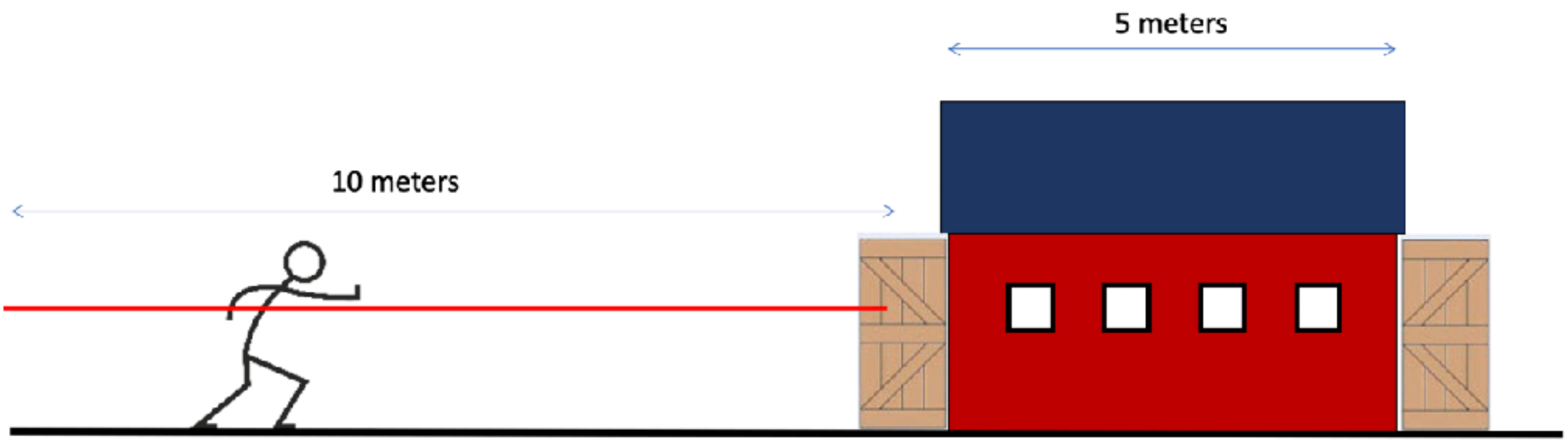
Picture a pole vaulter running towards a long barn (with doors at each end) at maximum speed. Recall from discussing time dilation/length contraction, the factor we are interested in is

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Let's pretend that the pole vaulter is *absurdly* fast and can run at v = 260,000,000 m/s = 0.866 c.

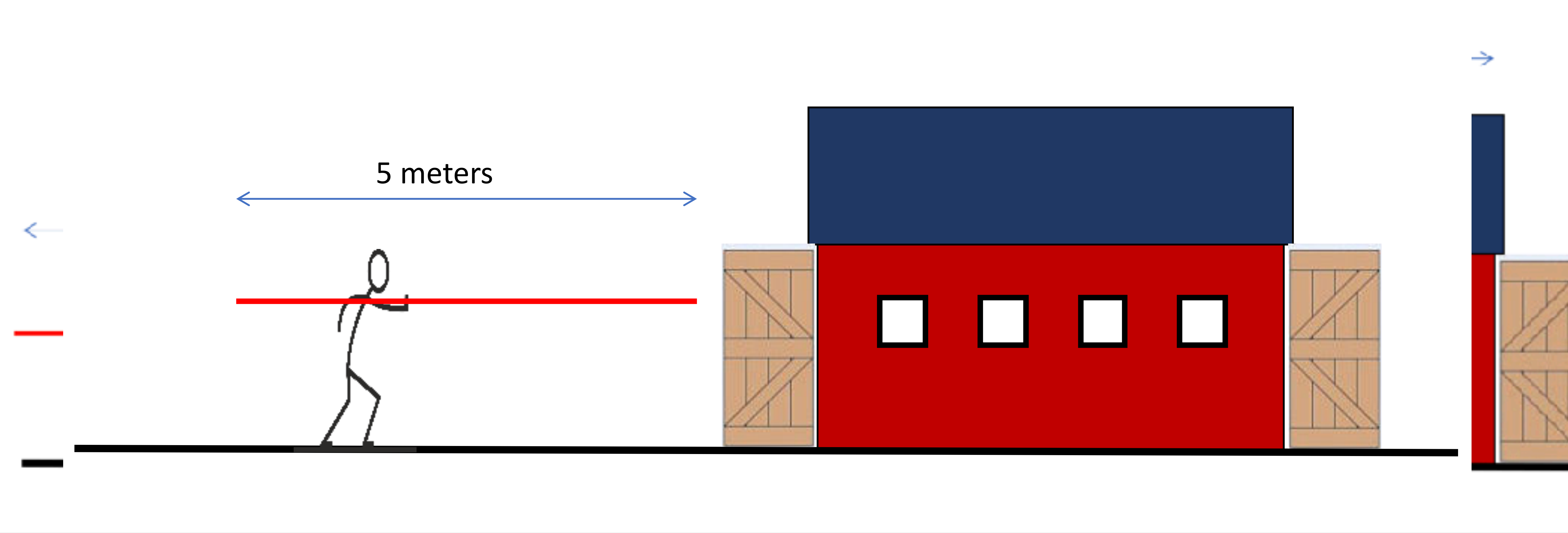
$$\gamma = \frac{1}{\sqrt{1 - (0.866)^2}} \approx 2$$

All lengths/times will be contracted/dilated by a factor of about 2. So, let's pretend that the barn is 5 m long* and the pole vaulter's pole is 10 m long*.



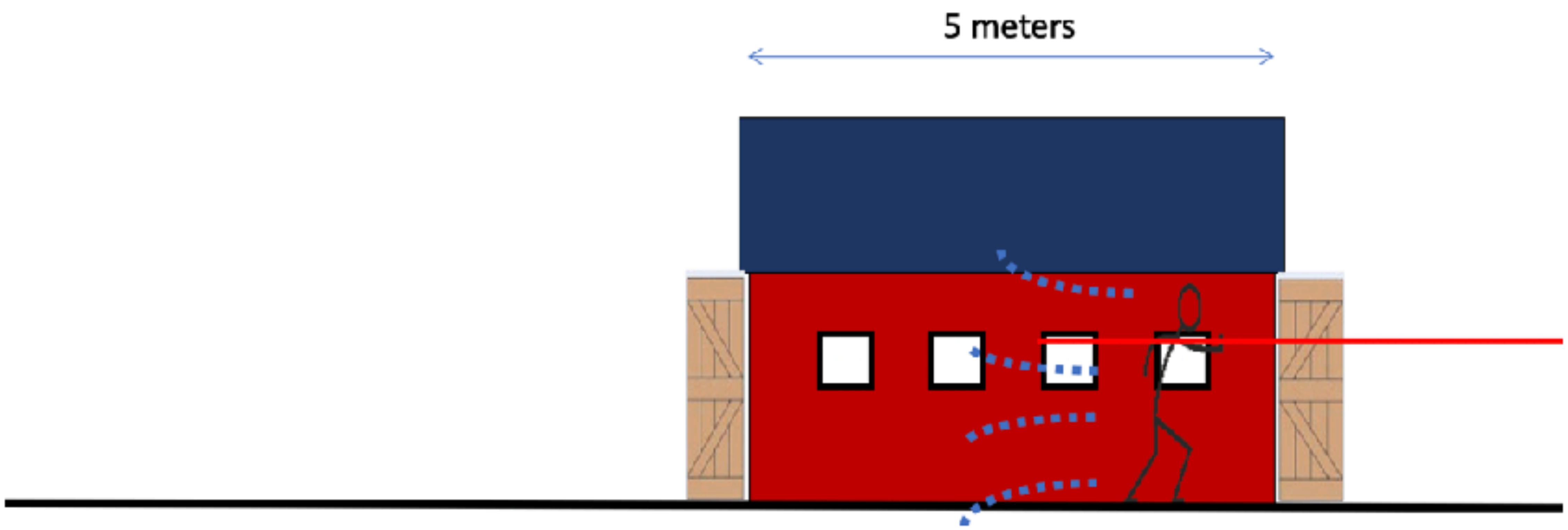
Pole Vaulter, as viewed by someone outside

- Because the pole vaulter is moving so fast, the pole appears to be shorter (as measured by us) than as measured by the pole vaulter. $$L' = \frac{L}{\gamma} = \frac{10\text{ m}}{2} = 5\text{ m}$$



So, as viewed by someone outside the barn, the pole is just as long as the barn is wide. Now, let's imagine that the pole vaulter runs into the barn…

Pole Vaulter Running…

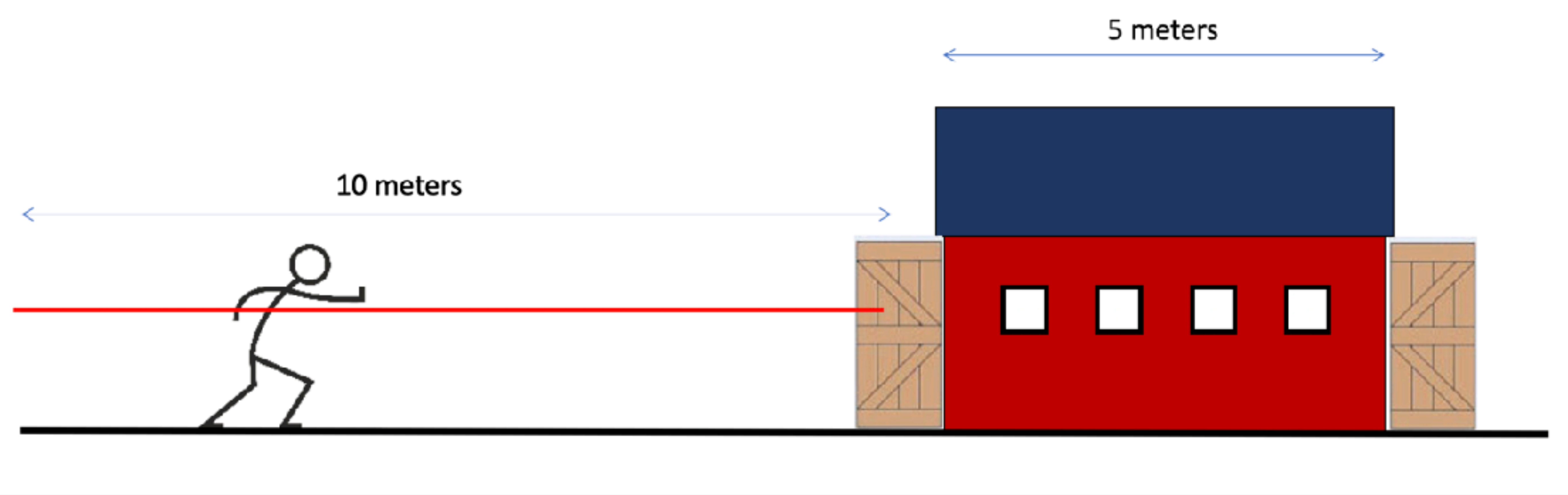


It seems that, because the pole is length contracted, the vaulter makes it through the barn with no problems.

Sitting outside, you have a magic button that you press (once the pole is completely inside) that slams both doors shut for the tiniest fraction of a second, then opens them up.

What about this whole situation from the Vaulter's Point of View?

- According to the vaulter, the pole is at rest (so it can't be length-contracted), but the barn is moving very, *very* fast towards him.



Lengths if everyone is at rest

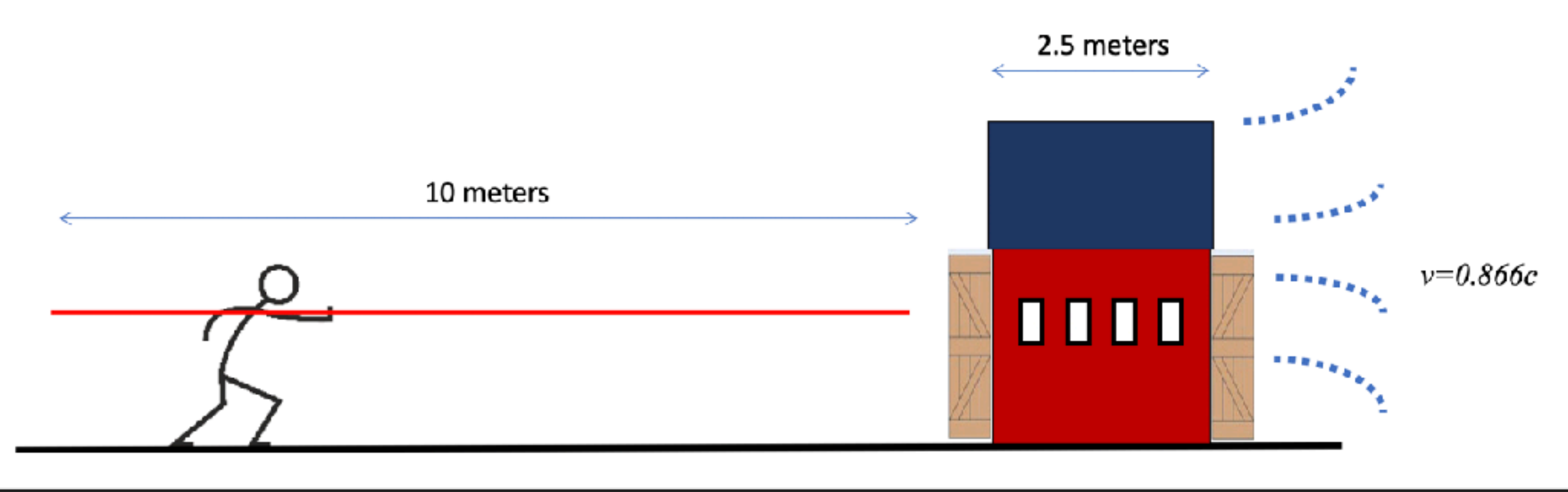
What do you think happens to the barn as viewed by the runner?

A) It grows longer

B) It shrinks shorter

C) It stays the same

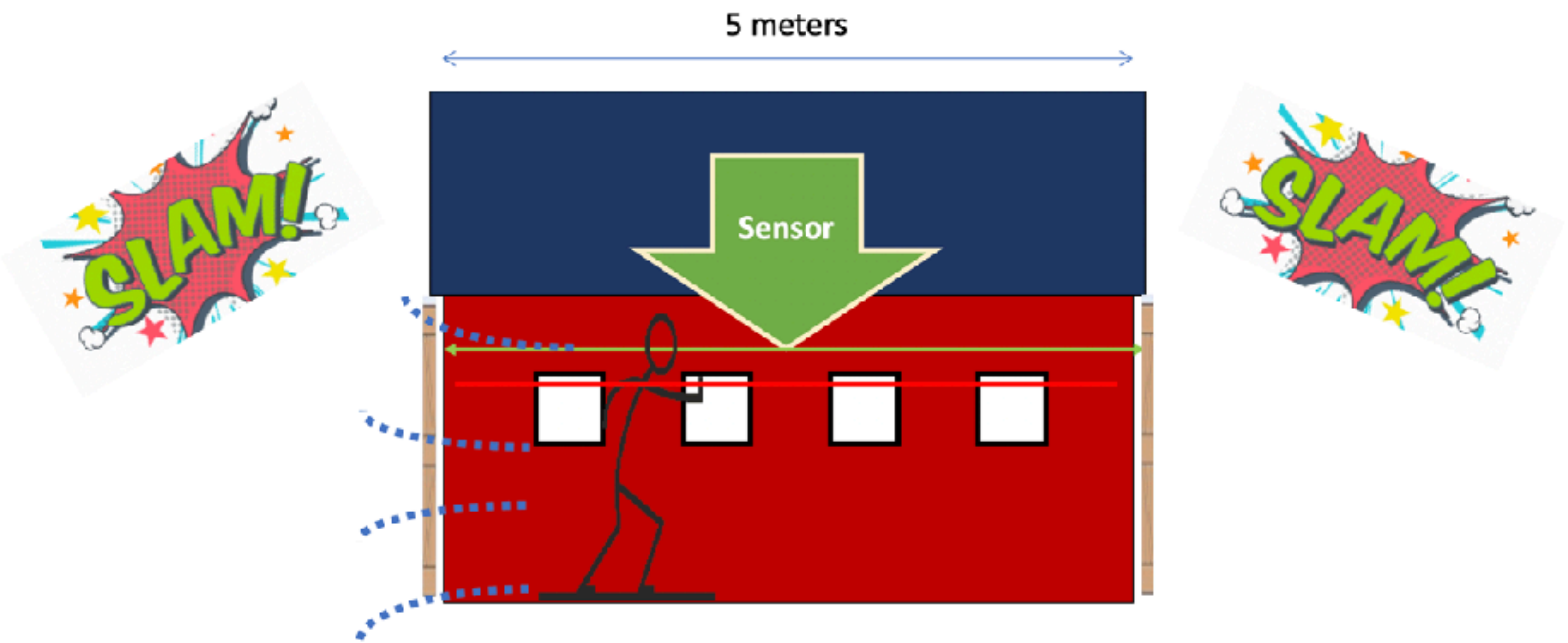
The Barn is Length-Contracted Too!



Let's resolve this "paradox". We have to apply both Length Contraction and Simultaneity.

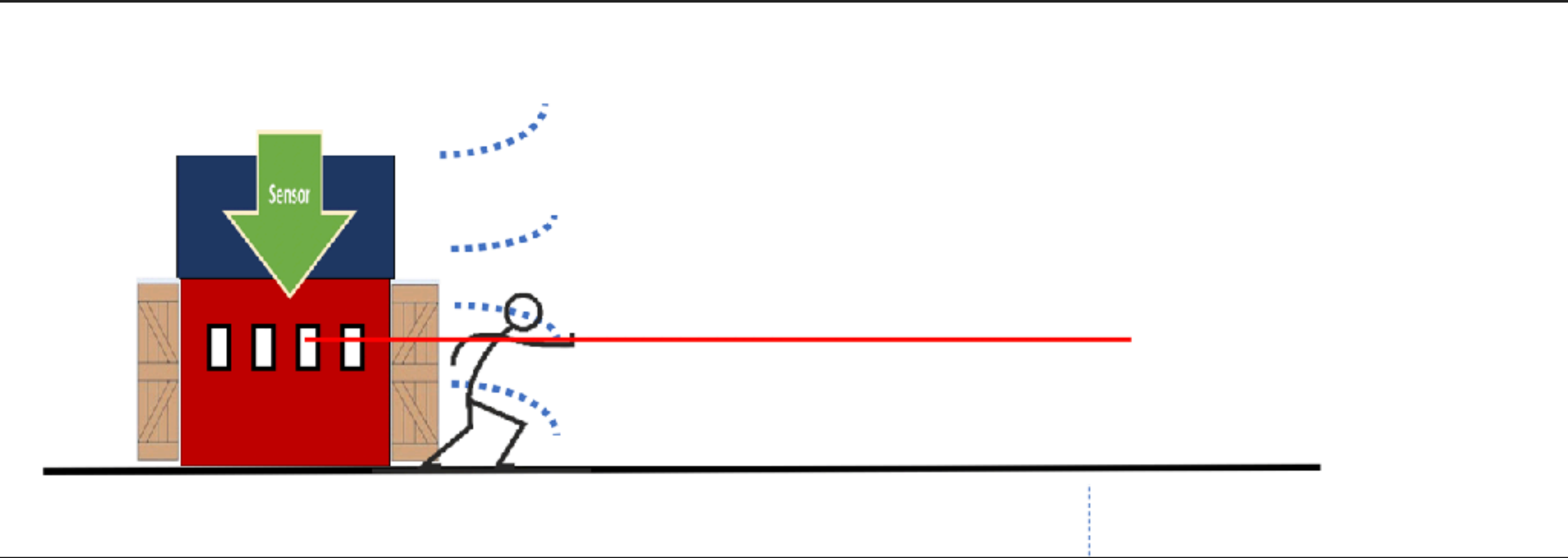
Back to the Observer Outside the Barn.

- What actually caused the barn doors to close (when the vaulter was inside)? We need a signal to travel to the doors and give them a ***simultaenous*** signal to close.



Recall, different observers can disagree about simultaneity!

Now, as viewed by the Pole Vaulter

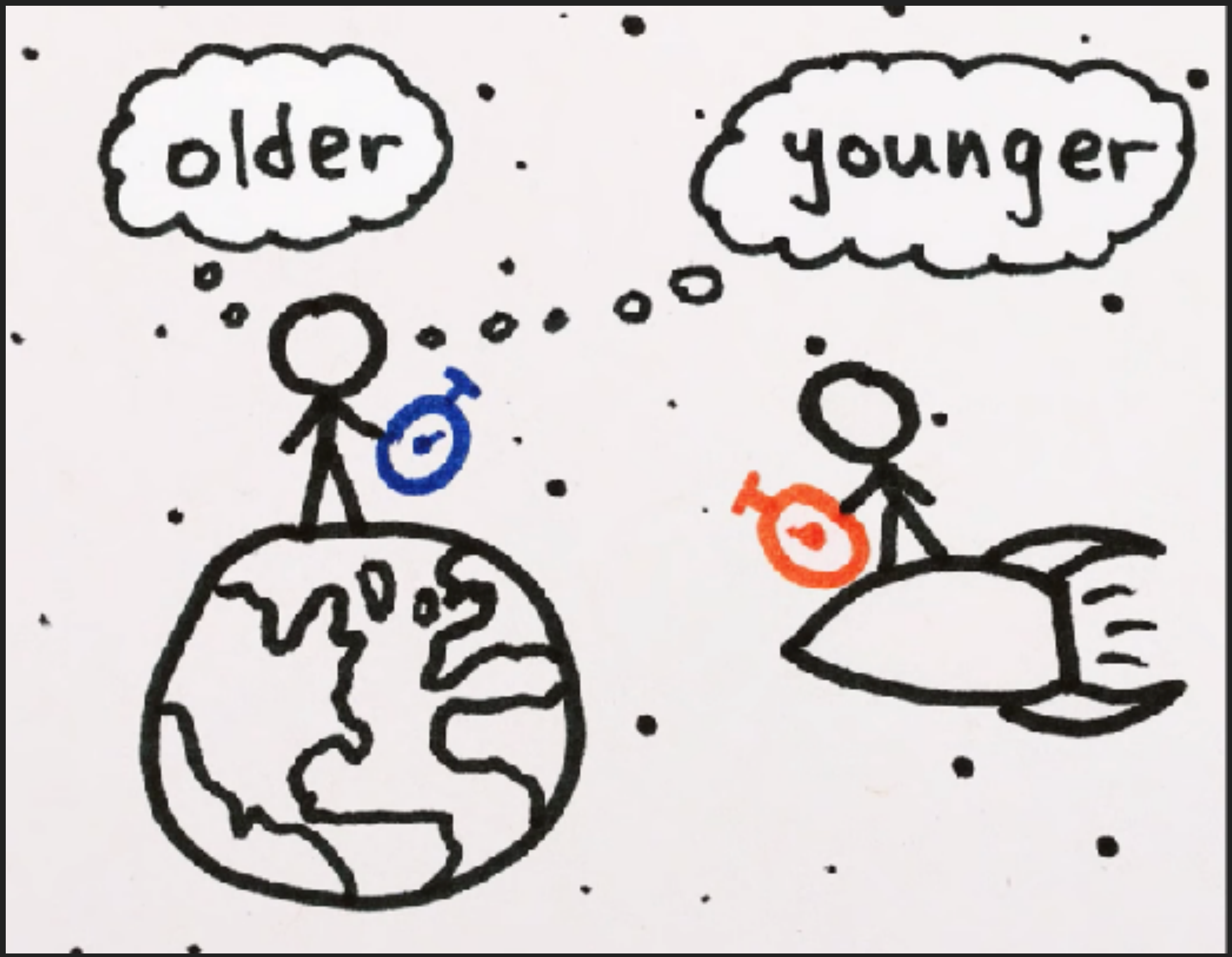


Just as according to the stationary observer, the pole vaulter makes it out unscathed. The two observers (stationary and the vaulter) may disagree on many things: the length of the pole, the length of the barn, the simultaneity of the doors closing, but they don't disagree on the impact of events: The vaulter made it through without breaking the pole.

More Strange "Paradoxes"

- Just like the pole vaulter and the barn, there are many examples of seemingly paradoxical situations that can be resolved if you're careful and follow the rules set forth by Special Relativity.

One particular favorite: "The Twin Paradox". One twin (Jimmy) stays at home, while the other (Janie) flies off in space, turns around, and returns. According to the Jimmy, Janie's clock runs slow, so Janie should be younger upon return. ***But***, from Janie's point of view, it was Jimmy who moved, so ***he*** should be younger once she returns. Who is correct?



https://www.youtube.com/watch?v=0iJZ_QGMLD0

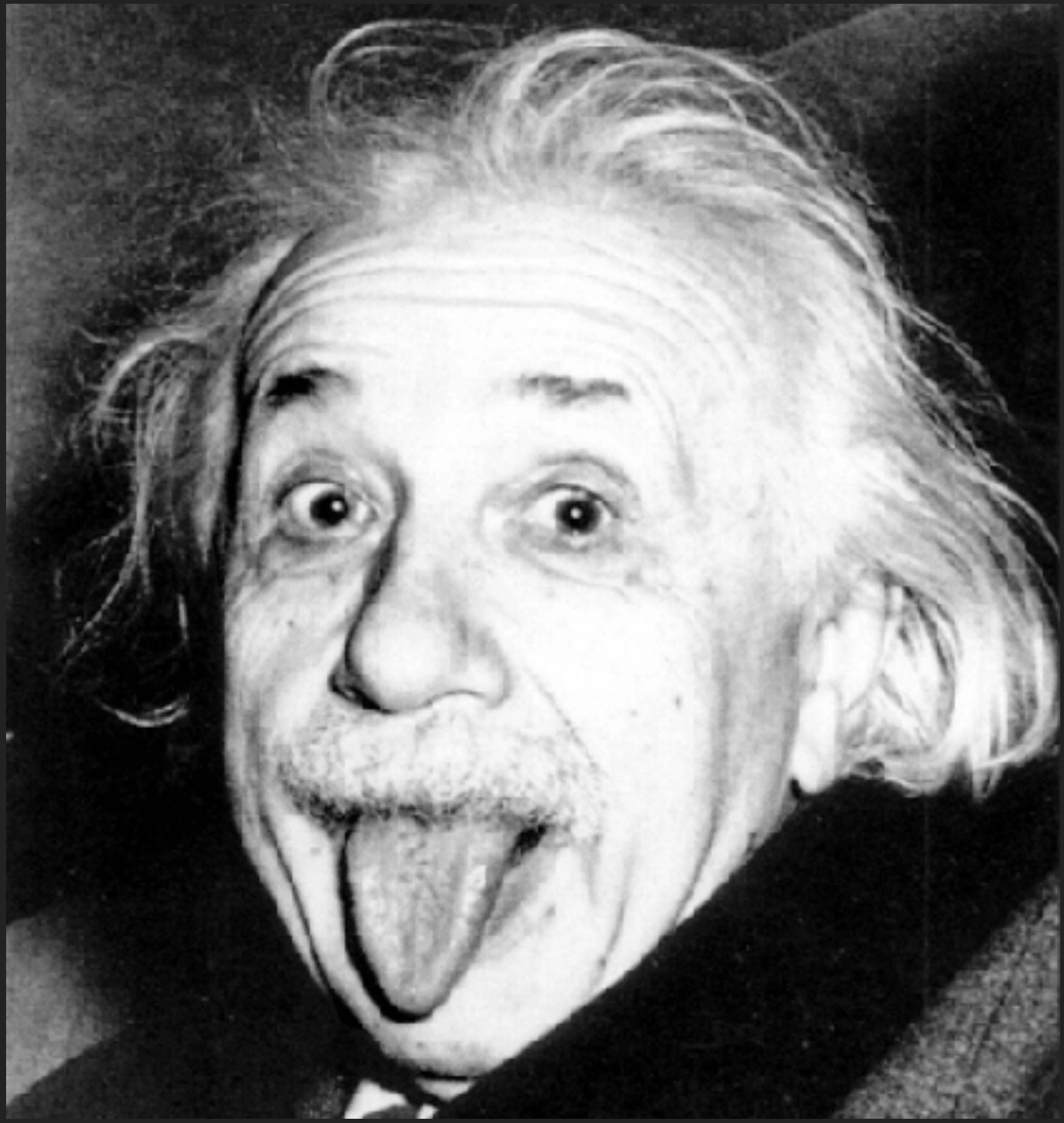
The relativistic roller coaster.

https://www.youtube.com/watch?v=9oM2NILrTC8

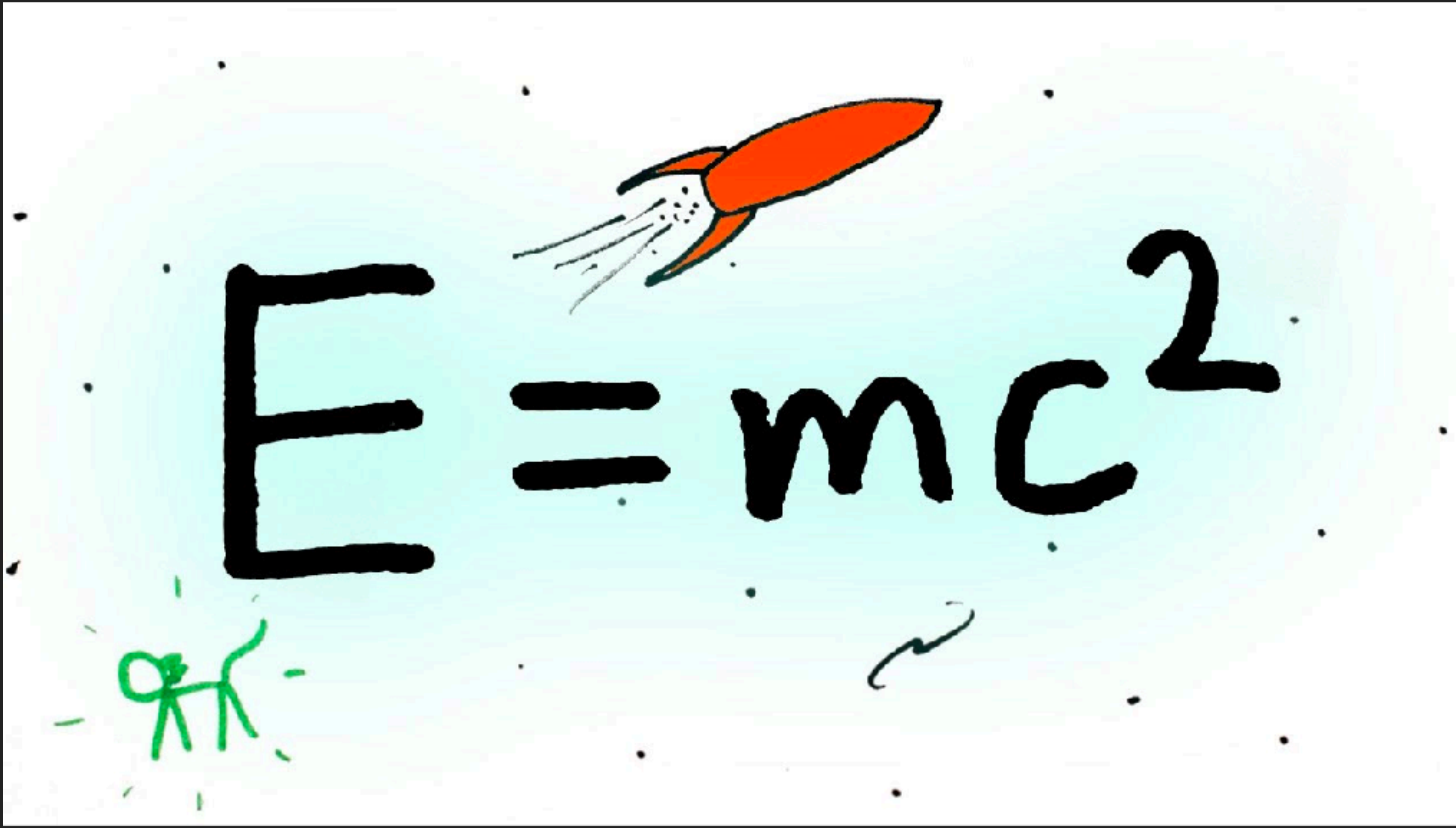
The creators of this video asked a simple question: What would happen if I rode a roller coaster in an alternate universe where the speed of light were much, much slower?

Relativistic Bike Ride

Back to Einstein's Miracle Year



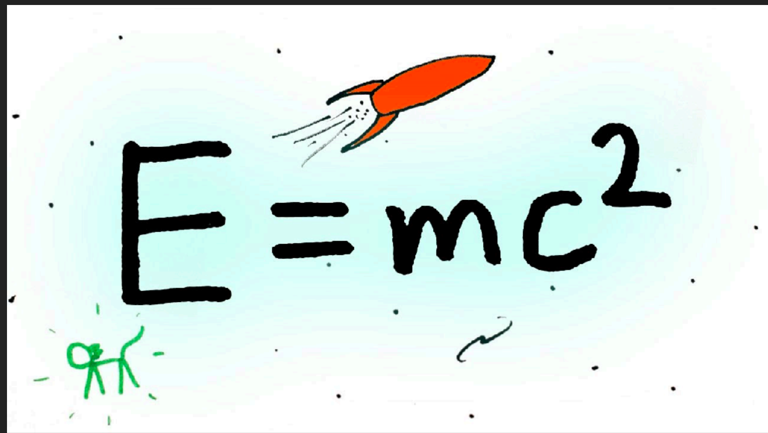
The Annus Mirabilis Papers



Einstein's Famous Equation

- Just like with lengths, times, and simultaneity, observers can disagree about the total energy an object has!
- If you've taken some physics, you may be familiar with the equation for kinetic energy, $K = \frac{1}{2}mv^2$
- Einstein showed that this is incomplete: we need to consider a *total* energy that is $E = \gamma mc^2$

This new version includes what's known as the "rest energy". Let's look at what this equation means for small speeds.



Einstein's Famous Equation

- Einstein showed that this is incomplete: we need to consider a *total* energy that is $E = \gamma mc^2$

$$E = \gamma mc^2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} mc^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} mc^2$$

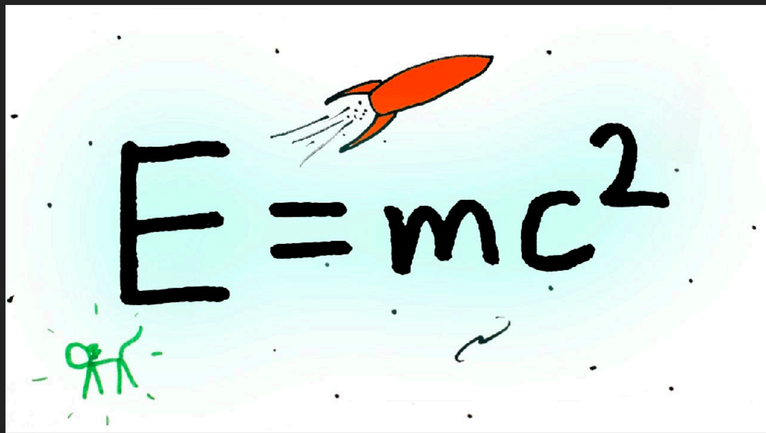
There's a rule in math (ask me after the talk!) that says if a number "x" is very, very small,

$$(1 + x)^y \approx 1 + y \times x$$

Applying this to our energy equation, we can see that

$$E \approx \left(1 + \frac{v^2}{2c^2}\right) mc^2 = mc^2 + \frac{1}{2} mv^2$$

The total energy of an object is its kinetic energy *plus* this rest energy!



How Big is this Rest Energy?

- Let's imagine my water bottle, which can hold about 1 liter (or 1 kilogram) of water.

- How much energy is stored in the "rest mass" of 1 kg of water?

$$E = mc^2 = \left(1\ \mathrm{kg}\right)\left(3\times 10^8\ \mathrm{m/s}\right)^2 = 9\times 10^{16}\ \frac{\mathrm{kg\ m^2}}{\mathrm{s^2}} = 9\times 10^{16}\ \mathrm{Joules}$$

Is that… a lot?

The most powerful explosion ever recorded, Tsar Bomba: 24×10^{16} Joules

Can anything move at the speed of light?

- Let's pretend we're in a rocket ship that's accelerating. Einstein's equation tells us that the energy of the rocket ship is $E = \gamma mc^2$, where m is the mass of the rocket.

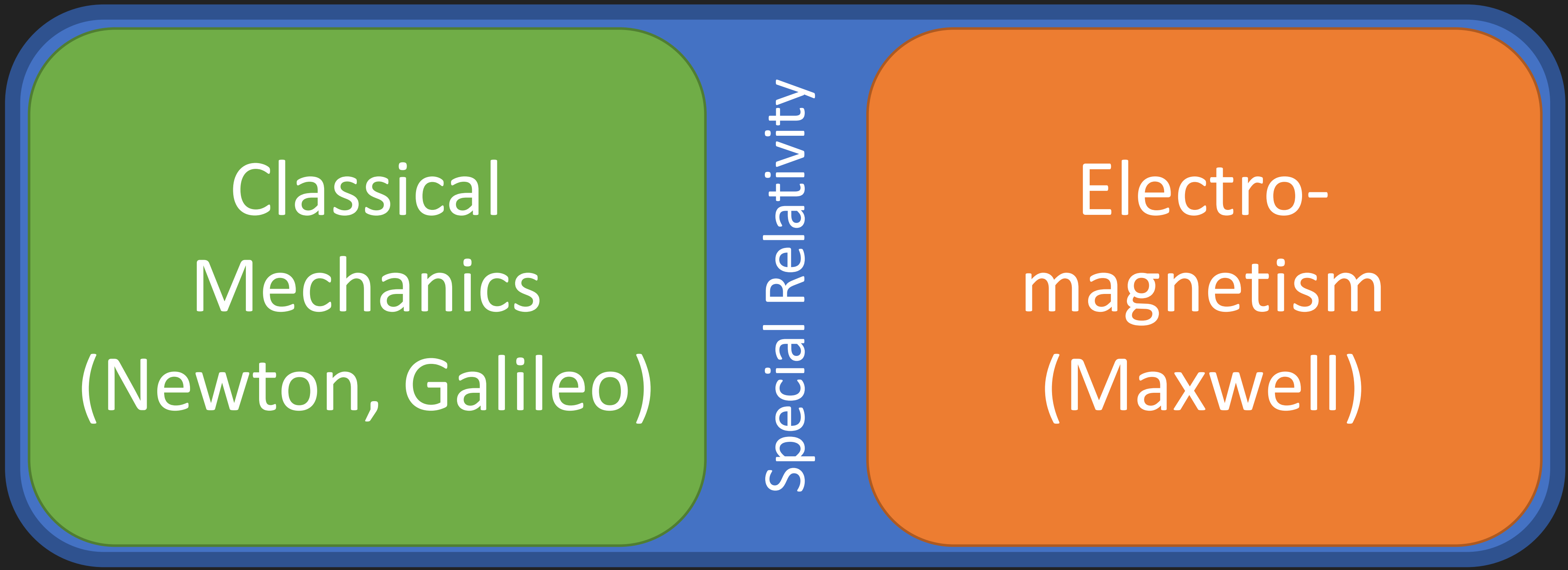
- If we keep accelerating and get closer to the speed of light, what happens to the factor γ?

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \longrightarrow \frac{1}{\sqrt{1 - \frac{c^2}{c^2}}} \rightarrow \frac{1}{\sqrt{1 - 1}}$$

Why should we believe Special Relativity is True?

- All of the examples I've shown require *very* fast speeds, or have *very* tiny effects. How can we be sure that this is a good description of nature?

- From a theoretical point of view, it's a "nice" theory because it contains and unites what we're familiar with, Classical Mechanics and Electricity/Magnetism



- Additionally, if you take the speed of objects to be slow (compared to lightspeed), everything looks like classical mechanics again!

Concrete Evidence of Special Relativity

- GPS Satellites work by using *very* precise clocks, accurate to within 20-30 nanoseconds. That corresponds to measuring locations of objects on the road to within about 5 meters of their actual location.

- To stay in orbit around Earth, GPS satellites have to move pretty fast – about 8,700 mph or 4,000 meters per second.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1.000000000088$$

Concrete Evidence of Special Relativity

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1.00000000088$$

- The moving clocks move slow (as measured by us on the ground). They appear to lose about 6 ns (relative to us) every minute.
- This seems small (within their accuracy), but it builds up!
- After one day, their clocks are off by over 7,000 ns – if their accuracy is that bad, they can't tell our position to any better than ~300 meters!

GPS Accuracy

Muon Decay and Cosmic Rays

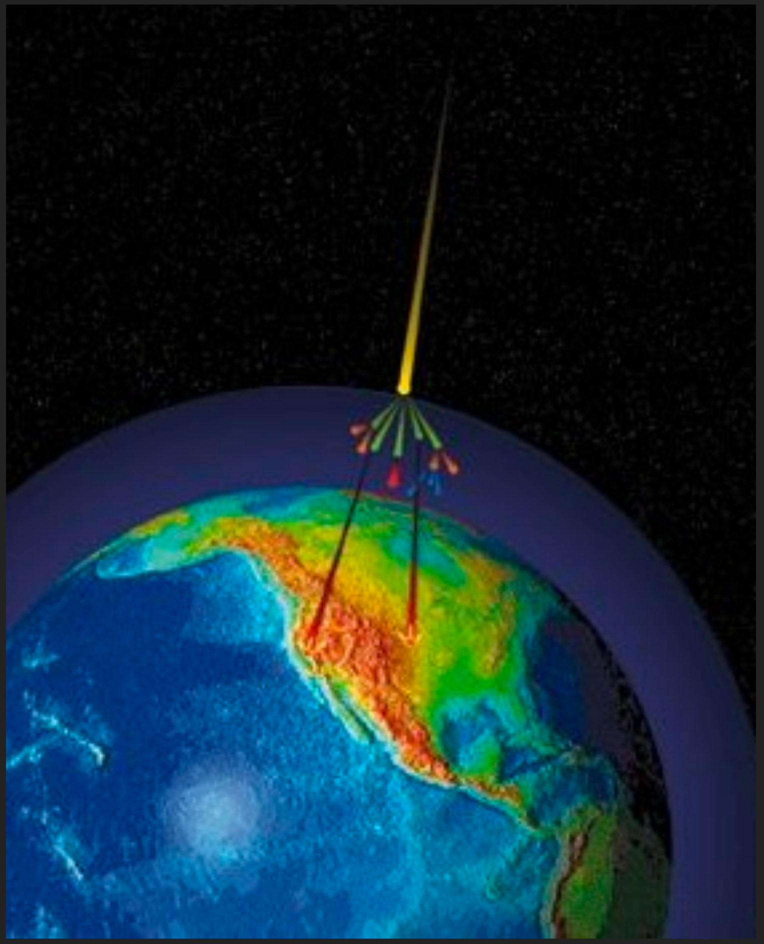
- Many particles in the standard model decay if we wait long enough, like the muon.
- The muon's ***half-life*** (half of all muons will have decayed by this time) is about 1.5 microseconds (0.0000015 s).
- Muons are constantly being produced when protons hit material in the upper atmosphere, and they shoot down toward the earth at roughly 98% the speed of light.

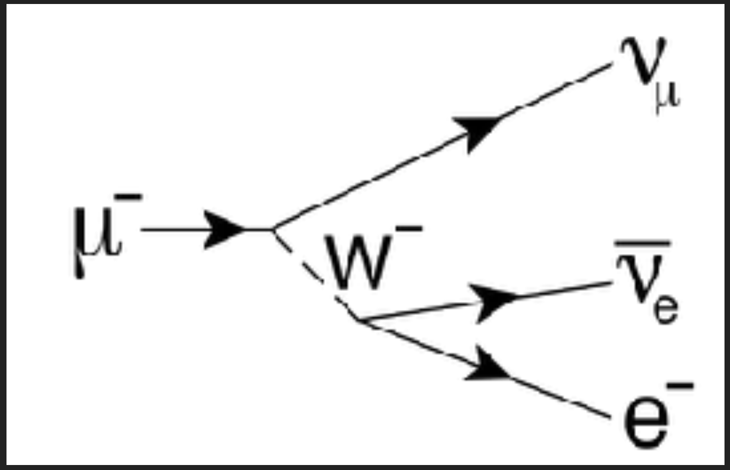
$$v = 0.98c = 2.94 \times 10^{8}\ \mathrm{m/s}$$

How far can a typical muon make it before decaying?

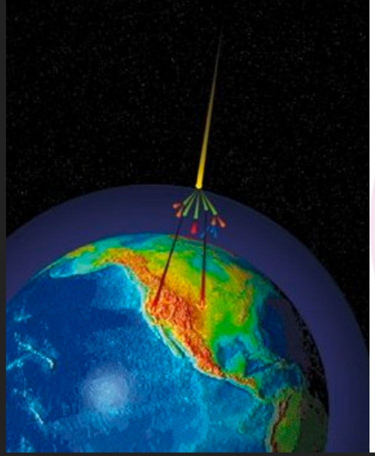
$$\text{distance} = \text{rate} \times \text{time}$$

$$d = \left(2.98 \times 10^{8}\ \mathrm{m/s}\right)\left(1.5 \times 10^{-6}\ \mathrm{s}\right) = 447\ \mathrm{m}$$

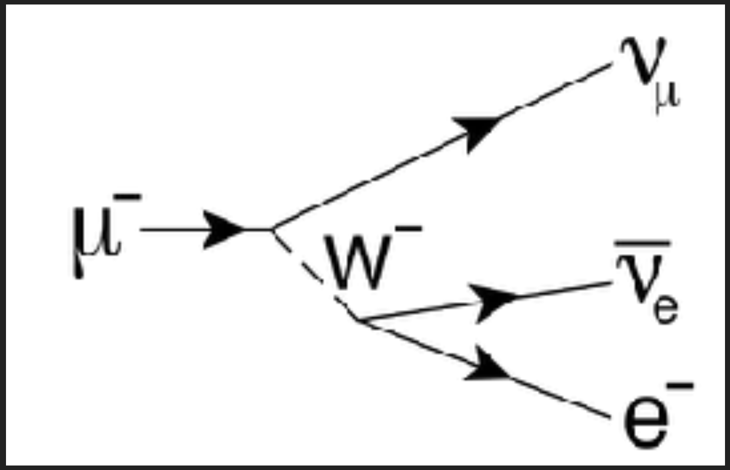




Muon Decay and Cosmic Rays



$$d = \left(2.98 \times 10^{8} \text{ m/s}\right)\left(1.5 \times 10^{-6} \text{ s}\right) = 447 \text{ m}$$



This says that for every 447 meters the muons travel, half of the remaining ones should decay. The muons are produced about 10,000 meters above the earth's surface, or about 22x times this distance.

$$N_{\text{Survive}} = N_{\text{Created}} \times 2^{-h/d}$$

Imagine we started with 1,000,000 muons in the upper atmosphere:

$$N_{\text{Survive}} = 1{,}000{,}000 \times 2^{-22} \approx 0.24$$

Less than one muon in a million should make it to Earth's surface before decaying!

We actually see about 49,000 muons per million reaching earth's surface…

Relativity to the Rescue!

- With the muons moving so fast, we should have considered relativistic effects. The half-life of a particle tells you how long a particle tends to take to decay *as measured by that particle*!

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 5$$

These muons have a pretty large boost factor!

Time Dilation Solution

If a muon measures a time to be 1.5 microseconds, we actually measure it to be dilated to 7.5 microseconds.

$$d = \left(2.98 \times 10^{8}\ \mathrm{m/s}\right)\left(7.5 \times 10^{-6}\ \mathrm{s}\right) = 2235\ \mathrm{m}$$

Instead of the 10,000 m the muons need to travel being 22 half-lives, this is now only about 4.5:

$$N_{\text{Survive}} = 1{,}000{,}000 \times 2^{-4.5} \approx 45{,}000$$

Length Contraction Solution

As viewed by the muon, the earth is rushing up at 98% the speed of light, and the 10,000 meters it needs to travel is actually length-contracted!

$$L = L_0/\gamma = 2{,}000\ \mathrm{m}$$

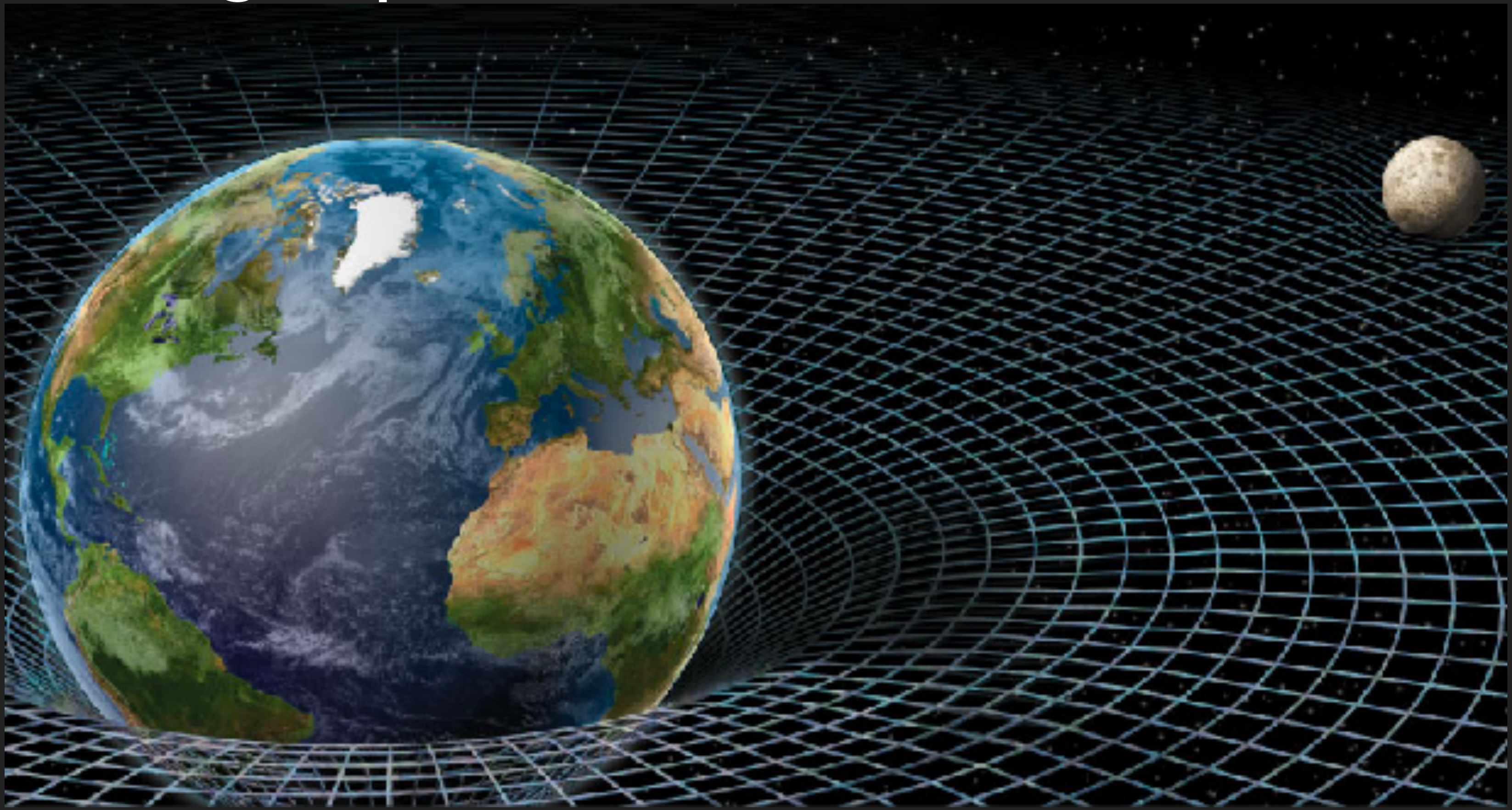
According to the muon, its half-life is still 1.5 microseconds, but now, 2,000 meters is only 4.5 half-lives:

$$N_{\text{Survive}} = 1{,}000{,}000 \times 2^{-4.5} \approx 45{,}000$$

Both answers get us much closer to the actual, observed value! Be careful not to apply both time dilation and length contraction, though.

Is Special Relativity the end of the story?

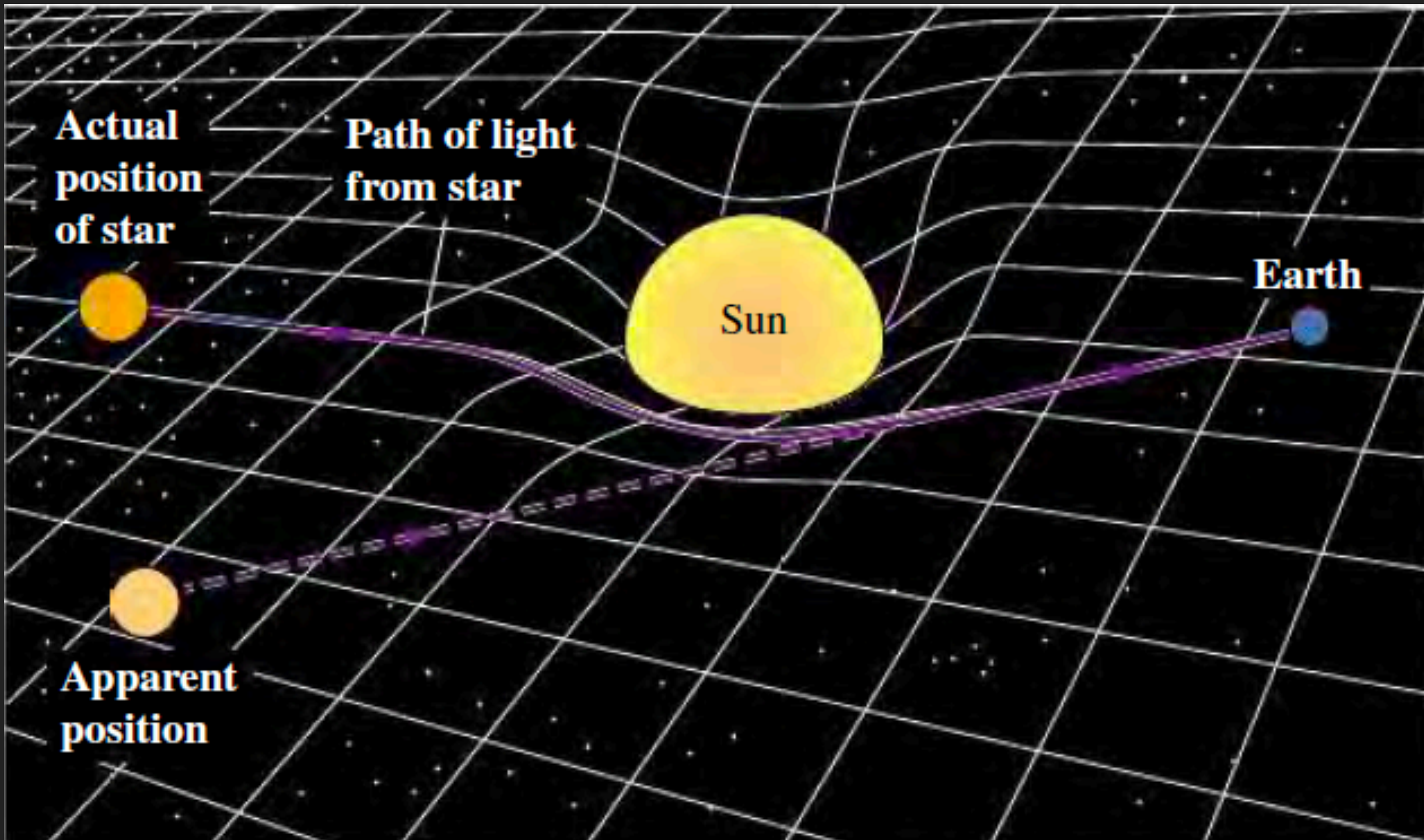
- Nope!
- Einstein furthered his theories to come up with General Relativity:
    - Space and time are distorted by the presence of mass/energy.
    - Masses move through space based on the curvature of spacetime.



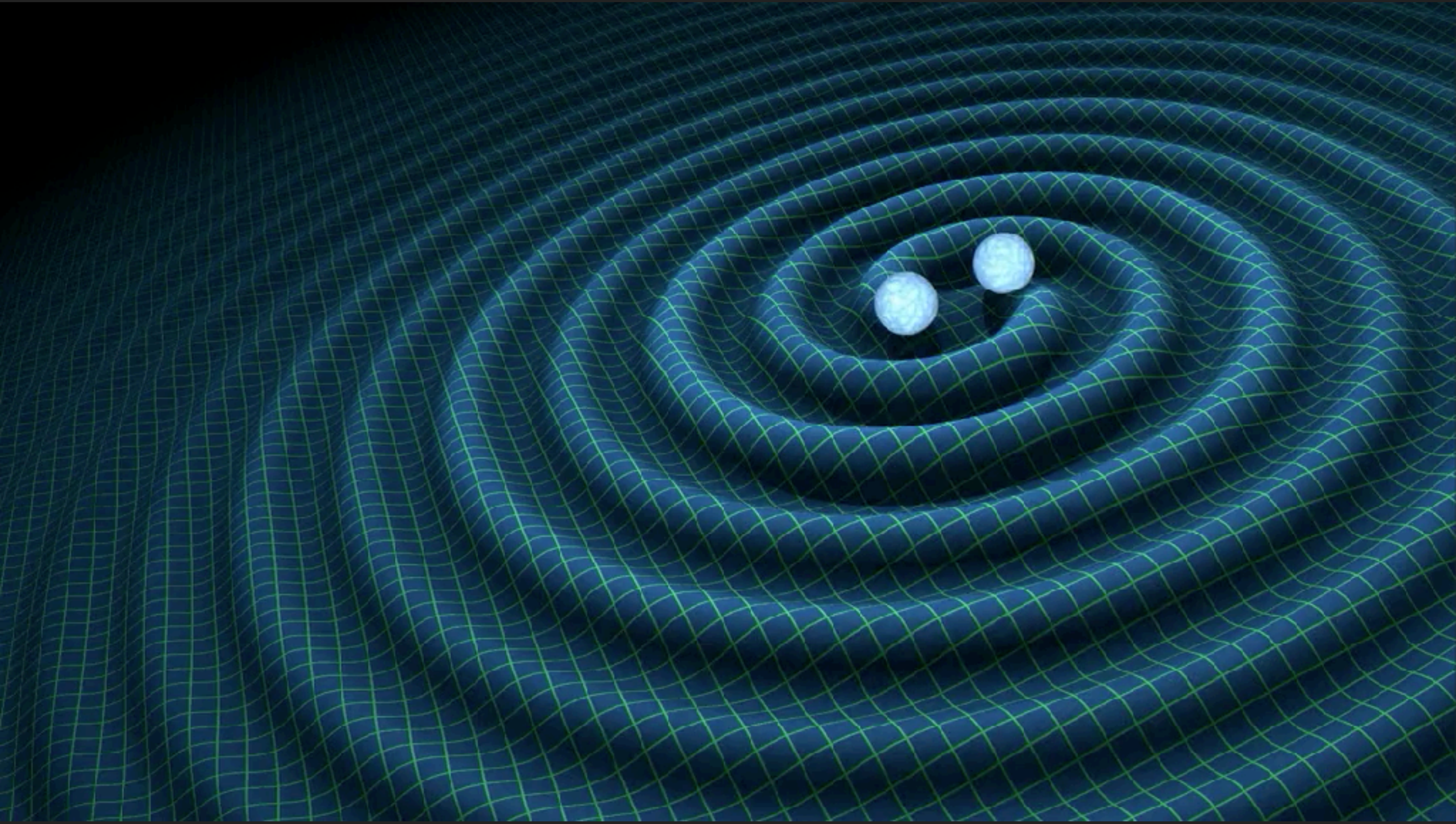
Amazing predictions from General Relativity

- Mercury's orbit precesses year-by-year
- The path of light is bent as it goes around massive objects
- Black holes exist – objects so dense that not even light can escape their gravity
- The universe is not "static", but it's actually expanding. 
- Objects moving through space can cause ripples in spacetime that travel away at the speed of light – gravitational waves. 

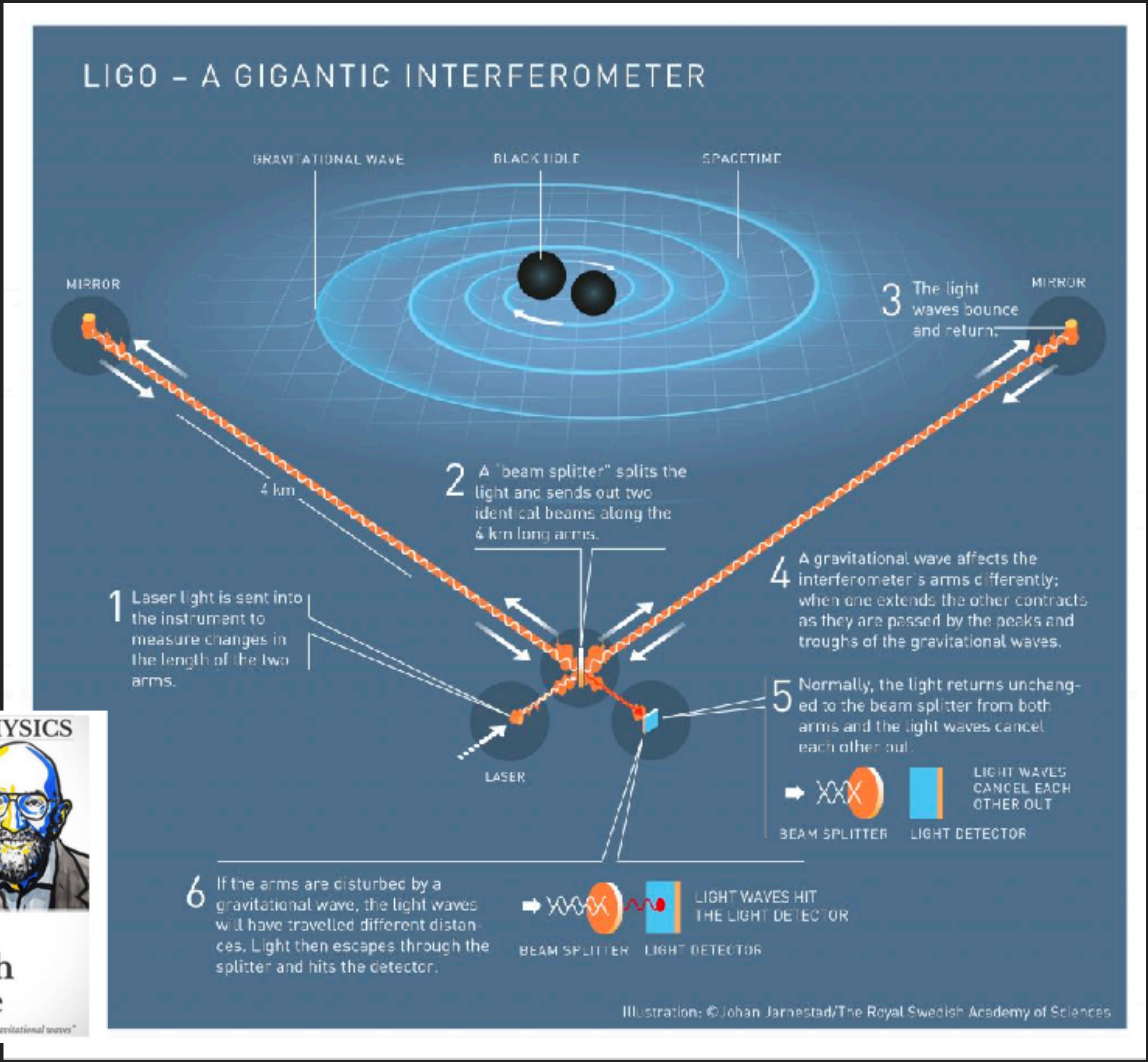
Light Bending around a Massive Star

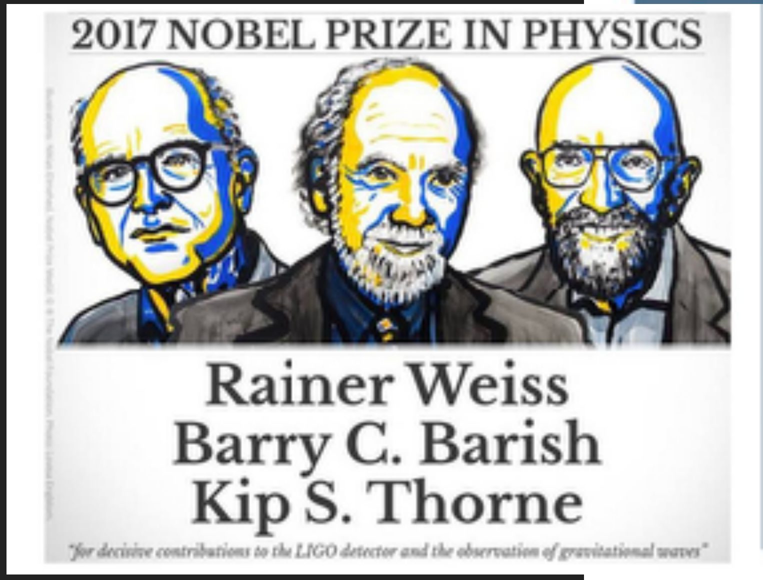


Gravitational Waves



Nobel Prize 2017





Recall – the Michelson-Morley experiment (looking for the aether) was also an interferometer!

To Conclude

- Some apparent inconsistencies in the late 1800s caused physicists to rethink some fundamental assumptions, like whether the speed of light can change from reference frame to reference frame.
- Special relativity causes disagreements: lengths, times, and orders of events are not always agreed upon by different observers!
- We may resolve these tensions with careful thought, however.
- Einstein went on to generalize this to General Relativity. His theories have revolutionized physics to this day.